GREENLEAF'S

NEW

INTELLECTUAL

ARITHMETIC.

• A

MENTAL ARITHMETIC,

ON THE

INDUCTIVE PLAN;

BEING AN

ADVANCED INTELLECTUAL COURSE,

DESIGNED FOR

♦

COMMON SCHOOLS AND ACADEMIES.

By BENJAMIN GREENLEAF, A.M.,
AUTHOR OF "NATIONAL ARITHMETIC," ETC.

IMPROVED ELECTROTYPE EDITION.

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PREFACE.

THE great benefits to be derived from the study of Mental
Arithmetic have, at length, become universally admitted.
This appreciation of the science has given rise to an urgent
demand for improved methods of teaching it. The mechani-
cal way of arranging a set of examples to a model, which
should give the pupil a solution of the whole, with little or
no mental effort, is no longer approved.

The object of this book is, therefore, to furnish a graded
course of lessons, fully up to the most approved standards
of instruction.

It has been the constant aim of the author, in its prepa-
ration, to unfold inductively the science of numbers in such
a series of progressive intellectual exercises as should
awaken latent thought, encourage originality, give activity
to invention, and develop the power of discriminating justly,
reasoning exactly, and applying readily results to practical
purposes.

Forms of analysis have been introduced throughout the
work, as a guide to the learner, but in connection with such
examples as shall, nevertheless, give proper scope to his
reasoning powers. It will be noticed, as a valuable original
feature of this work, that two forms of analysis are given
·in the first part of the book, one full, and the other abbre-
viated.

In the notes, aid is furnished more by hints and sugges-
tions than by full and formal solutions, which, if too numer-
ous, might discourage sufficiently persevering effort, and the
all-important habit of self-reliance.

Percentage and Interest receive full attention, and are treated, for the most part, in an original manner.

The advanced exercises in the fundamental processes of the science, given in the Appendix, constitute another feature peculiar to this work. These will be found not only useful as an intellectual drill, but also exceedingly valuable for preparing the learner to dispense with written operations in business life, to a far greater extent than has hitherto been deemed practicable.

The latter part of the Appendix, although quite brief, is intended to give the plan of the work, with additional models of analysis, and suggestions valuable alike to pupil and teacher.

SUGGESTIONS TO TEACHERS.

THE extent to which the book can be dispensed with by the class, in recitation, should be determined by the nature of the lesson and the attainments of the pupil.

When the book is not used, each question should be repeated by the pupil after the teacher, and the required solution should always be given promptly.

A full form of analysis should be insisted upon at first, but when it has become familiar, a more abbreviated one may be allowed.

No form of solution should be permitted to pass, unless it is neatly expressed, and is entirely accurate.

In general, when a pupil has thoroughly mastered the first fifty pages of this book, he may advantageously enter upon the study of the Common School Arithmetic, or the New Practical Arithmetic of Greenleaf's Series, and continue the intellectual course in connection with the written.

Classes in higher Arithmetic, and even in Algebra, may often be benefited by a review of the more difficult exercises of this book, in connection with those branches.

MENTAL ARITHMETIC.

LESSON I.

1. John had 1 peach, and his father gave him 1 more ; how many peaches did he then have ?

SOLUTION. — *Since John had 1 peach, and his father gave him 1 more, he then had 1 peach and 1 peach, which are 2 peaches:*

2. Susan has 2 books, and Mary has 1 book ; how many books have they both ?

3. If you had 2 cherries, and I should give you 2 more, how many cherries would you then have ?

4. Lucy found 2 pins, and Sarah found 3 pins ; how many did they both find ?

5. If you should recite 2 lessons to-day, and 4 more to-morrow, how many would you recite in all ?

6. A lemon cost 2 cents, and an orange cost 5 cents ; how many cents did both cost ?

7. Gave for a pencil 2 cents, and for some paper 6 cents ; what was the cost of both ?

8. On one bush there are 2 roses, and on another there are 7 roses ; how many on both bushes ?

9. 2 boys and 8 boys are how many boys ?

10. A farmer sold a lamb for 2 dollars, and a calf for 9 dollars ; how many dollars did he get for both ?

11. Alfred caught 3 birds, and Jason caught 1 bird ; how many birds did they both catch ?

12. James has 8 marbles, and Charles has 2 marbles ; how many marbles have they both ?

13.   A man sold a pig for 3 dollars, and a sheep for 3 dollars ; how many dollars did he receive for both ?

14.   Mary has 3 books, and Margaret has 4 books ; how many books have they both ?   .

15.   Edward gave 3 cents for a postage-stamp, and 5 cents for a box of wafers; how much did both cost ?

16.   Eliza is 3 years old ; how old will she be in 6 years, if she lives ?

17.   A farmer has 3 cows in one field, and 7 in another ; how many has he in both ?

18.   In a class there are 3 girls and 8 boys ; how many pupils are there in the class ?

19.   A boy found under one apple-tree 3 apples, and under another 9 apples ; how many did he find in all ?

20.   If you have 4 chestnuts in one hand, and 1 chestnut in the other, how many have you in both hands ?

21.   Susan had 4 merit marks, and obtained 2 more ; how many did she then have ?

22.   George found 4 eggs in one nest, and 3 eggs in another ; how many did he find in both ?

23.   A man bought a cord of wood for 4 dollars, and half a ton of coal for 4 dollars ; how much did both cost him ?

24.   A lady paid 4 cents for a skein of silk, and 5 cents for a spool of cotton ; how much did she pay for both ?

25.   Ella gave 4 cents for candy, and 6 cents for nuts ; how much did she give for both ?

26.   Lucy, having given to a beggar 4 cents, found she then had 7 cents left ; how many cents had she at first ?

27.   Alfred bought a hook for 4 cents, and a line for 8 cents ; how much did both cost ?

28.   A farmer sold 4 cows, and then had 9 left; how many cows had he at first ?

LESSON II.

1. 1 and 1 are how many ?

ANSWER. 1 *and* 1 *are* 2.

2. 2 and 1 are how many ?
3. 2 and 2 are how many ?
4. 2 and 3 are how many ?
5. 2 and 4 are how many ?
6. 2 and 5 are how many ?
7. 2 and 6 are how many ?
8. 2 and 7 are how many ?
9. 2 and 8 are how many ?
10. 2 and 9 are how many ?
11. 3 and 3 are how many ?
12. 3 and 4 are how many ?
13. 3 and 5 are how many ?
14. 3 and 6 are how many ?
15. 3 and 7 are how many ?
16. 3 and 8 are how many ?
17. 3 and 9 are how many ?
18. 4 and 4 are how many ?
19. 4 and 5 are how many ?
20. 4 and 6 are how many ?
21. 4 and 7 are how many ?
22. 4 and 8 are how many ?
23. 4 and 9 are how many ?

24. Abby found 5 pins, and Jane found 1 more ; how many did they both find ?

25. Ellen had 5 chickens, and her father gave her 2 more ; how many did she then have ?

26. Mary gave 5 cents for tape, and 3 cents for thread ; how much did she give for both ?

27. George bought 5 marbles, and had 4 given him ; how many did he then have ?

28. John gave to one schoolmate 5 nuts, and to another the same number ; how many did he give to both ?

29. Olive had 5 pins on her cushion, and stuck on it 6 more; how many did she then have?

30. If you spend 5 cents, and have 7 cents left, how many had you at first?

31. 5 oranges and 8 oranges are how many oranges?

32. Joseph, having lost 5 cents, had only 9 cents left; how many had he at first?

33. Gave 6 cents for paper, and 1 cent for a pen; how much did both cost?

34. If you had 6 apples, and should have 2 more given you, how many would you then have?

35. How many slates are 6 slates and 3 slates?

36. Gave 6 cents for paper, and 4 cents for quills; how many cents were paid for both?

37. If you should give 6 dollars for a vest, and 5 dollars for a pair of boots, how much would both cost?

38. A bookseller sold in one day 6 books, and in another day 6 more; how many did he sell in all?

39. A farmer sold 6 sheep, and retained 7; how many had he at first?

40. If a clock cost 6 dollars, and a table 8 dollars, what would be the cost of both?

41. A farmer has 6 cows in one pasture, and 9 in another; how many has he in both?

42. Paid 7 cents for a ruler, and 1 cent for a pencil; what did both cost?

43. If a paper of pins cost 7 cents, and a pencil 2 cents, how many cents must be paid for both?

44. If 7 birds are upon a gate, and 8 upon the ground, how many are there in all?

45. 7 books and 4 books are how many books?

46. 7 horses and 5 horses are how many horses?

47. Laura had 7 needles, and her sister gave her 6 more; how many did she then have?

LESSON III.

1. 5 and 5 are how many?
2. 5 and 6 are how many?
3. 5 and 7 are how many?
4. 5 and 8 are how many?
5. 5 and 9 are how many?
6. 6 and 6 are how many?
7. 6 and 7 are how many?
8. 6 and 8 are how many?
9. 6 and 9 are how many?
10. 7 and 3 are how many?
11. 7 and 4 are how many?
12. 7 and 2 are how many?
13. 5 and 4 are how many?
14. 7 and 1 are how many?
15. 1 and 9 are how many?
16. 6 and 4 are how many?

17. William bought 7 marbles, and had 7 more given him; how many did he then have?

18. If Charles has two notes due him, one for 7 dollars, and the other for 8 dollars, how much is due him in all?

19. There are on one side of a room 7 chairs, and on the other side 9; how many are there on both sides?

20. John had 8 cents, and his father gave him 1 more; how many cents did he then have?

21. James is 8 years old; how old will he be if he lives 2 years longer?

22. Paid 8 dollars for sugar, and 3 dollars for salt; how much was paid for both?

23. In a certain class there are 8 boys, and 4 girls; how many scholars are there in the class?

24. Gave 8 dollars for wood, and 5 dollars for coal; how much did both cost?

1 *

25. If a coat cost 8 dollars, and a vest 6 dollars, how much will both cost ?

26. If you had 8 dollars, and your father should give you 7 more, how many would you have ?

27. If a barrel of flour cost 8 dollars, and half a barrel of beef cost 8 dollars, how much will·both cost ?

28. Bought a hundred weight of sugar for 8 dollars, and a quantity of butter for 9 dollars ; what did the whole cost ?

29. Levi earned in one week 9 dollars, and in another week only 1 dollar ; how much did he earn in all ?

30. Paid 9 dollars for a saddle, and 2 dollars for a bridle ; how much did both cost ?

31. Andrew paid 9 cents for a quart of nuts, and 3 cents for candy ; how much did the whole cost him ?

32. James raised 9 melons, and his brother raised 4 ; how many did they both raise ?

33. There are 9 apples on one table, and 5 on another ; how many are there on both tables ?

34. If you should have 9 pens given you of one kind, and 6 of another kind, how many pens would you then have ?

35. On one side of a pond are 9 lilies, and on the other 6 ; how many are there on both sides ?

36. Edward walked 9 miles in one day, and 7 miles the next day ; how many· miles did he walk in all ?

37. A farmer sold 9 pounds of butter at one time, and 8 pounds at another ; how many pounds did he sell in all ?

38. A miller ground 9 bushels of wheat, and 9 bushels of corn ; how many bushels did he grind in all ?

LESSON IV.

1. 2 and 8 are how many?
2. 2 and 6 are how many?
3. 6 and 8 are how many?
4. 3 and 5 are how many?
5. 5 and 5 are how many?
6. 4 and 3 are how many?
7. 3 and 9 are how many?
8. 9 and 2 are how many?
9. 7 and 7 are how many?
10. 1 and 3 are how many?
11. Three and six are how many?
12. Four and five are how many?
13. Five and one are how many?
14. Three and eight are how many?
15. Nine and three are how many?
16. Nine and four are how many?
17. Eight and two are how many?
18. Five and seven are how many?
19. One and eight are how many?
20. Two and two are how many?
21. Eight and six are how many?
22. Two and seven are how many?
23. Nine and five are how many?
24. Two and three are how many?
25. Three and four are how many?
26. Four and two are how many?
27. How many are four and seven?
28. How many are four and eight?
29. How many are four and nine?
30. How many are six and nine?
31. How many are six and six?
32. How many are eight and three?
33. How many are eight and eight?
34. How many are eight and seven?

LESSON V.

ADDITION is the process of finding the *sum* of two or more numbers. The result obtained is also called the *amount*.

1. John has 3 marbles, Samuel has 5, and Jacob has 4 ; how many have they all ?

SOLUTION. — *Since John has 3 marbles, Samuel 5, and Jacob 4, they all have the sum of 3 marbles, 5 marbles, and 4 marbles ; 3 and 5 are 8, and 4 are 12. Therefore they all have 12 marbles.*

2. Gave 3 cents to Susan, 4 to Emily, and 2 to Ann ; how many cents were given to them all ?

3. Gave 4 nuts to one boy, 2 to another, and 4 to another ; how many nuts were given to the three boys ?

4. Bought a pound of sugar for 9 cents, a pound of raisins for 7 cents, and an ounce of nutmegs for 6 cents ; what was the cost of the whole ?

5. James is 4 years old, Edward 6, and Charles 8 ; what is the sum of their ages ?

6. Bought a sheep for 9 dollars, a lamb for 2 dollars, and a pig for 5 dollars ; what did the whole cost ?

7. Paid for wood 7 dollars, for coal 6 dollars, and for a saw 2 dollars ; how much did the whole cost ?

8. Charles had 3 peaches, and received 3 more from Albert, and 2 more from Edmund ; how many did he then have ?

9. A farmer sold 6 bushels of wheat, 7 bushels of rye, and 8 bushels of corn ; how many bushels did he sell ?

10. A lady expended for silk 4 dollars, for gloves 1 dollar, and for a bonnet 9 dollars ; how many dollars did she expend in all ?

11. Bought a barrel of flour for 6 dollars, a barrel of apples for 3 dollars, and a keg of molasses for 8 dollars ; what was the cost of the whole ?

12. John spends 9 cents for paper, 3 cents for wafers, and 5 cents for pens; how many cents does he spend in all?

13. George buys 6 oranges at one time, 2 at another, and 8 at another; how many oranges does he buy in all?

14. 5 dollars, 9 dollars, and 4 dollars are how many dollars?

15. How many are 1, 7, and 6? 3, 5, and 4? 3, 4, and 2? 4, 5, and 6?

16. Susan has 3 birds, Abby 4, and Ellen 9; how many have they all? 3, 4, and 9 are how many?

17. How many pigeons are 10 pigeons and 2 pigeons? 10 pigeons and 3 pigeons? 10 pigeons and 4 pigeons?

18. Gave 10 cents for a pine-apple, 6 cents for a ball, and 4 cents for a ruler; how many cents were given for the whole?

19. How many are 9, 7, and 6? 4, 8, and 6? 9, 2, and 5? 7, 4, and 7?

20. Paid for a coat 10 dollars, for a hat 5 dollars, and for a vest 3 dollars; how much was paid for the whole?

21. How many are 10 and 5? 10 and 6? 10 and 7? 10 and 9? 10 and 8?

22. If your brother should give you 10 apples, your uncle 4 apples, and your father 3 apples, how many apples would you then have?

23. Three boys, James, Henry, and Charles, went a fishing; James caught 10 fishes, Henry 8, and Charles 6; how many did they all catch? 10, 8, and 6 are how many?

24. Lucy bought some pins for 10 cents, some thread for 9 cents, and some tape for 4 cents; how much did they all cost? 10, 9, and 4 are how many?

LESSON VI.

1. How many are 1 and 10 ? 1 and 20 ? 1 and 30 ? 1 and 40 ? 1 and 50 ? 1 and 60 ? 1 and 70 ? 1 and 80 ? 1 and 90 ?

2. How many are 1 and 11 ? 1 and 22 ? 1 and 33 ? 1 and 44 ? 1 and 55 ? 1 and 66 ? 1 and 77 ? 1 and 88 ? 1 and 99 ?

3. How many are 2 and 11 ? 2 and 21 ? 2 and 31 ? 2 and 41 ? 2 and 51 ? 2 and 61 ? 2 and 71 ? 2 and 81 ? 2 and 91 ?

4. How many are 2 and 9 ? 2 and 19 ? 2 and 29 ? 2 and 39 ? 2 and 49 ? 2 and 59 ? 2 and 69 ? 2 and 79 ? 2 and 89 ? 2 and 99 ?

5. How many are 3 and 8 ? 3 and 18 ? 3 and 28 ? 3 and 38 ? 3 and 48 ? 3 and 58 ? 3 and 68 ? 3 and 78 ? 3 and 88 ? 3 and 98 ?

6. How many are 4 and 3 ? 4 and 13 ? 4 and 23 ? 4 and 33 ? 4 and 43 ? 4 and 53 ? 4 and 63 ? 4 and 73 ? 4 and 83 ? 4 and 93 ?

7. How many are 5 and 5 ? 5 and 15 ? 5 and 25 ? 5 and 35 ? 5 and 45 ? 5 and 55 ? 5 and 65 ? 5 and 75 ? 5 and 85 ? 5 and 95 ?

8. How many are 6 and 4 ? 6 and 14 ? 6 and 24 ? 6 and 34 ? 6 and 44 ? 6 and 54 ? 6 and 64 ? 6 and 74 ? 6 and 84 ? 6 and 94 ?

9. How many are 7 and 6 ? 7 and 16 ? 7 and 26 ? 7 and 36 ? 7 and 46 ? 7 and 56 ? 7 and 66 ? 7 and 76 ? 7 and 86 ? 7 and 96 ?

10. How many are 8 and 7 ? 8 and 17 ? 8 and 27 ? 8 and 37 ? 8 and 47 ? 8 and 57 ? 8 and 67 ? 8 and 77 ? 8 and 87 ? 8 and 97 ?

11. How many are 9 and 4 ? 9 and 14 ? 9 and 24 ? 9 and 34 ? 9 and 44 ? 9 and 54 ? 9 and 64 ? 9 and 74 ? 9 and 84 ? 9 and 94 ?

12. How many are 6 and 6 ? 6 and 16 ? 6 and

26? 6 and 36? 6 and 46? 6 and 56? 6 and 66?
6 and 76? 6 and 86? 6 and 96?

13. How many are 3 and 5? 3 and 15? 3 and
25? 3 and 35? 3 and 45? 3 and 55? 3 and 65?
3 and 75? 3 and 85? 3 and 95?

14. How many are 7 and 3? 7 and 13? 7 and
23? 7 and 33? 7 and 43? 7 and 53? 7 and 63?
7 and 73? 7 and 83? 7 and 93?

15. How many are 2 and 6? 2 and 16? 2 and
26? 2 and 36? 2 and 46? 2 and 56? 2 and 66?
2 and 76? 2 and 86? 2 and 96?

16. How many are 3 and 3? 3 and 13? 3 and
23? 3 and 33? 3 and 43? 3 and 53? 3 and 63?
3 and 73? 3 and 83? 3 and 93?

17. How many are 8 and 8? 8 and 18? 8 and
28? 8 and 38? 8 and 48? 8 and 58? 8 and 68?
8 and 78? 8 and 88? 8 and 98?

18. How many are 8 and 9? 8 and 19? 8 and
29? 8 and 39? 8 and 49? 8 and 59? 8 and 69?
8 and 79? 8 and 89? 8 and 99?

19. How many are 9 and 10? 9 and 20? 9 and
30? 9 and 40? 9 and 50? 9 and 60? 9 and 70?
9 and 80? 9 and 90?

20. How many are 10 and 10? 10 and 20? 10
and 30? 10 and 40? 10 and 50? 10 and 60? 10
and 70? 10 and 80? 10 and 90?

21. How many are 9 and 9? 9 and 19? 9 and
29? 9 and 39? 9 and 49? 9 and 59? 9 and 69?
9 and 79? 9 and 89? 9 and 99?

22. How many are 10 and 11? 10 and 21? 10
and 31? 10 and 41? 10 and 51? 10 and 61? 10
and 71? 10 and 81? 10 and 91?

23. How many are 10 and 12? 10 and 22? 10
and 32? 10 and 42? 10 and 52? 10 and 62? 10
and 72? 10 and 82? 10 and 92?

24. How many are 9 and 11? 9 and 31? 9
and 43? 9 and 86? 9 and 97?

LESSON VII.

A UNIT is a single thing, or one.

A CONCRETE NUMBER is a number whose units have reference to some particular thing or quantity; as, 15 cents, 3 men.

An ABSTRACT NUMBER is a number whose units have no reference to any particular thing or quantity; as, 5, 3.

A SIGN is a symbol employed to indicate the relations of quantities, or operations to be performed upon them.

The SIGN of EQUALITY is two short horizontal lines, =, and signifies *equal to;* thus, 1 foot = 12 inches, is read, 1 foot is equal to 12 inches.

The SIGN of Addition is an erect cross, +, which signifies *plus, and,* or *added to ;* thus, 4 + 6 = 10, is read, 4 plus 6 are equal to 10.

1. Charles gave 15 cents for an arithmetic, 10 cents for a grammar, and 8 cents for a writing-book; what was the cost of the whole?

2. 15 + 10 + 8 are how many?

3. Bought 12 bales of cotton, 6 bags of rice, and 2 boxes of sugar; what is the number of articles purchased?

4. 12 + 6 + 2 are how many?

5. A lady purchased some silk for 25 dollars, and a shawl for 5 dollars; how much did she give for both?

6. A man bought a cow for 35 dollars, and a calf for 5 dollars; how much did both cost?

7. 35 + 5 are how many?

8. If a wagon is worth 50 dollars, a saddle 9 dollars, and a whip 1 dollar, what is the value of the whole?

9. A boy found 62 chestnuts under one tree, and 20 under another; how many did he find in all?

10. 62 + 10 + 10 are how many?

11. 34 + 26 + 6 are how many?

12. A gentleman gave 46 dollars for a watch, 7 dollars for a chain, and two dollars for a key; how many dollars did he pay for the whole?

13. $46 + 7 + 2$ are how many?

14. Lucy had 70 pins in a cushion, and put in 20 more; how many had she then in the cushion?

15. I gave 80 apples to Peter, and had 20 apples left; how many did I have at first?

16. $60 + 20$ are how many?

17. $35 + 20 + 15$ are how many?

18. New York has 59 counties, Delaware has 3, and Rhode Island 5; how many counties have the three States?

19. A man gave 64 dollars for a piece of land; it cost him 10 dollars to fence it, and 2 dollars to have it plowed; what was the whole cost?

20. A farmer raised 40 bushels of oats, 50 bushels of corn, and 20 bushels of turnips; how many bushels in all did he raise?

21. Rufus received 40 cents on his birthday, and 40 cents at Christmas; how many cents did he receive in all?

22. A farmer kept his sheep in 4 pens; in the first there were 20, in the second there were 10, in the third there were 8, and in the fourth there were 6; how many sheep did he have?

23. My book-case has 4 shelves; the first shelf contains 16 books, the second 12 books, the third 7 books, and the fourth 6 books; how many books are there in the book-case?

24. A man started on a journey; the first day he traveled 30 miles, the second day 10 miles, and the third day 9 miles; how many miles did he travel?

25. Edward bought a vest for 98 cents, some buttons for 12 cents, and some thread for 6 cents; what was the whole cost?

26. $83 + 17 + 3 + 7$ are how many?

27. $22 + 18 + 25 + 3$ are how many?

28. What examples of this lesson contain abstract numbers? What ones concrete?

LESSON VIII.

SUBTRACTION is the process of taking one number from another, to find the *difference*. The result obtained is also called the *remainder*.

1. If you have 2 apples, and give 1 of them to your brother, how many apples will you have left?

SOLUTION. — *If I have 2 apples, and give 1 of them away, I shall have left the difference between 2 apples and 1 apple, which is 1 apple.*

2. James had 3 pencils, and gave 1 away; how. many pencils did James then have?

3. Lucy had 4 books, and gave 2 of them to Jane; how many books did she then have?

4. If I have 5 peaches, and should eat 3 of them, how many should I have left?

5. Charles had 6 doves, but the cat caught 3 of them; how many did he then have?

6. Rufus caught 7 fishes, and threw 4 of them back into the water; how many had he left?

7. Lydia had 8 nuts, but has eaten 4 of them; how many has she left?

8. I had 9 sheets of paper, but have given 5 of them to Charles; how many have I left?

9. William had 8 pears, but has given 6 of them to his teacher; how many pears has he left?

10. William caught 10 squirrels, but 6 of them were allowed to escape; how many were retained?

11. Thomas had 12 cents, but has spent 7 of them; how many has he left?

12. Gave 13 dollars for a barrel of flour, and 8 dollars for a tub of butter; how much more did the flour cost than the butter?

13. A farmer sold 16 sheep, and 8 lambs; how many more sheep did he sell than lambs?

14. James had 13 chickens, but 6 were taken by the hawks; how many remained?

15. George planted 19 trees, but only 9 lived; how many died?

16. Gave 15 cents for oranges, and 9 cents for lemons; how much more did the oranges cost than the lemons?

17. Paid 20 cents for nails, and 10 cents for brads; how much did the one cost more than the other?

18. Bought a clock for 10 dollars, and sold it for 15 dollars; what was gained?

19. John found 12 eggs, and Samuel 8; how many more did John find than Samuel?

20. A man engaged to labor 17 days, but left at the end of 11 days; how many more days had he agreed to labor?

21. Gave 11 dollars to Emily, and 7 dollars to Betsey; how many more dollars were given to Emily than to Betsey?

22. Paid 20 cents for a penknife, and 12 cents for an inkstand; how much more did the penknife cost than the inkstand?

23. Sarah is 17 years old, and Isabel is 9 years old; what is the difference of their ages?

24. A farmer raised 25 bushels of beans, and 11 bushels of peas; how many more bushels of beans did he raise than of peas?

25. Henry had 35 pins, and lost 15 of them; how many had he left?

26. Mary went a shopping with 40 cents in her purse, and when she returned she had only 10 cents remaining; how many cents had she spent?

27. I had 14 oranges, and sold 7 of them; how many had I left?

28. Thomas had 18 birds, and 9 of them flew away; how many birds remained?

29.   Henry had 12 quarts of berries, and sold 6 of them ; how many had he left ?

. 30.   Thomas recited 25 perfect lessons, and William only 8 ; how many more did Thomas recite than William ?

31.   Bought a cow for 27 dollars, and sold her for 24 dollars ; how much did I lose by the bargain ?

32.   Sold a lot of wood for 20 dollars, and received in payment some cloth worth 7 dollars ; how much was still due ?

33.   Bought a wagon for 28 dollars, and sold it for 40 dollars ; how much was gained by the bargain ?

34.   40 less 28 are how many ?

35.   A man paid 14 dollars to one man, and 4 to another ; how much more did he pay to one than the other ?

36.   From a vessel containing 40 gallons, 15 gallons leaked out ; how many gallons still remained ?

37.   A boy counted his chickens one night, and found he had 19 ; he counted them the next morning, and found he had but 14 ; how many were missing ?

38.   A horse traveled 40 miles one day, and 27 the next day ; how many more miles did he travel the first than the second day ?

39.   Bought a carriage for 60 dollars, and a harness for 35 dollars ; how much more did the carriage cost than the harness ?

40.   A cistern, which holds 90 gallons, was full in the morning, but at night there wore but 30 gallons left ; how many gallons had leaked out ?

41.   If a man is now 50 years old, what was his age 24 years ago ?

42.   John is 11 years old, and his father 40 ; how many years younger is John than his father ?

43.   How many are 80 less 62 ?

LESSON IX.

1.   4 less 2 are how many ?
     ANS. 4 *less* 2 *are* 2.
2.   4 less 3 are how many ?
3.   5 less 3 are how many ?
4.   6 less 4 are how many ?
5.   7 less 5 are how many ?
6.   7 less 4 are how many ?
7.   8 less 6 are how many ?
8.   9 less 5 are how many ?
9. 10 less 9 are how many ?
10. 11 less 8 are how many ?
11. 13 less 9 are how many ?
12. 12 less 7 are how many ?
13. 14 less 6 are how many ?
14. 15 less 3 are how many ?
15. 16 less 6 are how many ?
16. 19 less 9 are how many ?
17. 21 less 10 are how many ?
18. 25 less 10 are how many ?
19. 29 less 9 are how many ?
20. 24 less 8 are how many ?
21. 30 less 10 are how many ?
22. 42 less 10 are how many ?
23. 24 less 10 are how many ?
24. 30 less 15 are how many ?
25. 20 less 5 are how many ?

The SIGN of SUBTRACTION is a short horizontal line, —, which signifies *minus*, or *less;* thus, 8 — 6 = 2 is read, 8 minus 6 are equal to 2.

26. How many are 24 — 12?
27. How many are 30 — 8?
28. How many are 40 — 2?
29. How many are 40 — 7?
30. How many are 50 — 7?

31. How many are $60 - 10$?
32. How many are $37 - 10$?
33. How many are $49 - 19$?
34. How many are $33 - 13$?
35. How many are $23 - 10$?
36. How many are $43 - 10$?
37. How many are $44 - 14$?
38. How many are $34 - 4$?
39. How many are $34 - 14$?
40. How many are $53 - 23$?
41. How many are $50 - 9$?
42. How many are $60 - 20$?
43. How many are $70 - 20$?
44. How many are $90 - 30$?
45. How many are $80 - 40$?
46. How many are $90 - 40$?
47. How many are $92 - 40$?
48. How many are $70 - 30$?
49. How many are $75 - 25$?
50. How many are $77 - 27$?
51. How many are $67 - 42$?
52. How many are $72 - 42$?
53. How many are $87 - 33$?
54. How many are $89 - 14$?
55. How many are $94 - 64$?
56. How many are $97 - 43$?
57. How many are $79 - 31$?
58. How many are $86 - 43$?
59. How many are $57 - 26$?
60. How many are $81 - 31$?
61. How many are $85 - 45$?
62. How many are $92 - 12$?
63. How many are $96 - 16$?
64. How many are $100 - 20$?
65. How many are $100 - 25$?

LESSON X.

1.ˑ A farmer sold 10 sheep to one man, 6 to another, and had 12 left; how many had he at first?

2. $10 + 6 + 12$ are how many?

3. A man had 16 dollars; he paid 4 to one man, and 6 to another; how many dollars were left?

4. $16 - 4 - 6$ are how many?

5. Sold a chest of tea for 25 dollars, which was 7 dollars more than it cost; how much did it cost?

6. Paid 12 dollars for a barrel of flour, and 9 dollars for a hundred-weight of sugar; what was the cost of both, and how much more was paid for the flour than for the sugar?

7. John had 29 apples; he gave 9 to his brother, 7 to his sister, and the rest to his mother; how many did he give to his mother?

8. $29 - 9 - 7$ are how many?

9. James bought a slate for 15 cents, and some pencils for 6 cents; if he had at first 30 cents, how much more has he to spend?

10. Isabel is 25 years younger than her mother, who is 45 years old; what is Isabel's age?

11. How much less than 40 is $25 + 10$?

12. How much less than 48 is $31 + 9$?

13. How much greater than 45 is $59 - 9$?

14. How much greater than 50 is $63 - 3$?

15. How much greater than 63 is $87 - 4$?

16. How much more than $40 + 13$ is 60?

17. How much less than 75 is $35 + 20$?

18. John found that, if he had caught 7 fishes more, he should have caught 50 fishes; how many did he catch?

19. Bought one barrel of pork for 20 dollars, and another for 16 dollars, and sold the whole for 40 dollars; how much was the gain?

20. George spent 19 cents for candy, and 21 cents

for fruit; how much more would he have to spend to make 50 cents?

21.   Thirty-two, and eight, and five, less ten, are how many?

22.   Twenty-eight, and three, and nine, less eight, are how many?

23.   Forty-one, and six, and three, and nine, less seven, are how many?

24.   Sixty-four, and four, and six, and one, less five, are how many?

25.   Nineteen, and eleven, and seven, less twenty, are how many?

26.   Fifty-nine, and nine, and seven, and five, less thirty, are how many?

27.   Seventy-seven, and seven, and six, and four and eight, less two, are how many?

28.   Eighty-six, and fourteen, and twenty, less five, are how many?

29.   Ninety-nine, and eleven, and ten, and five, less twenty, are how many?

30.   Two men bought a horse, the one paying 90 dollars, and the other 80 dollars less; how many dollars did both pay?

31.   Henry, who is 19 years old, is 10 years older than James, and 3 years older than Arthur; required the ages of James and Arthur.

32.   Sarah, 4 years ago, was 15 years old, and Mary is now 18 years old; what is the difference in their ages?

33.   James found under one tree 20 apples, under another 13 apples, and under another 8 apples; how many apples did he have left after giving his brother 11 of them?

34.   If you should earn 80 cents one day and spend 20 cents, and the next day your father should give you 30 cents, how much money would you then have?

LESSON XI.

MULTIPLICATION is the process of taking one number as many times as there are units in another number. The result obtained is called the *product.*

1.   What cost 2 apples, at 2 cents apiece?

ANALYSIS.   FULL FORM.   *2 apples will cost 2 times as much as 1 apple; then, if 1 apple cost 2 cents, 2 apples will cost 2 times 2 cents, which are 4 cents.   Therefore 2 apples, at 2 cents apiece, will cost 4 cents.*

ABBREVIATED FORM. — *Since 1 apple costs 2 cents, 2 apples will cost 2 times 2 cents, which are 4 cents.*

2.   What cost 2 lemons, at 3 cents apiece?

3.   What cost 2 oranges, at 4 cents apiece?

4.   What cost 2 oranges, at 5 cents apiece?

5.   What cost 2 pounds of rice, at 6 cents a pound?

6.   Bought 2 writing-books, at 7 cents apiece; what did they cost?   .

7.   If 1 pound of sugar costs 8 cents, what will 2 pounds cost?

8.   If 1 pound of veal costs 9 cents, what will 2 pounds cost?

9.   If there are 10 trees on each side of the street, how many are there on both sides?

10.   What cost 2 quarts of berries, at 11 cents a quart?

11.   What cost 2 pine-apples, at 12 cents apiece?

12.   What cost 3 pears, at 4 cents apiece?

13.   What cost 3 quarts of milk, at 5 cents a quart?

14.   What cost 3 yards of braid, at 6 cents a yard?

15.   If a horse will trot 7 miles in one hour, how far will he trot in 3 hours?

16.   If I give 8 cherries for 1 apple, how many cherries must I give for 3 apples?

17.   Bought 3 yards of cloth, at 9 cents a yard; how much did it cost?

18.   If a ship sails 10 miles in 1 hour, how far will it sail in 3 hours?

19.   If 3 boys have 11 marbles apiece, how many have they in all?

20.   What cost 3 loaves of bread, at 12 cents a loaf?

21.   What cost 4 bushels of cranberries, at 3 dollars a bushel?

22.   At 4 dollars a yard, what cost 4 yards of broadcloth?

23.   What cost 5 vests, at 4 dollars apiece?

24.   At 4 cents a pound, what cost 6 pounds of rice?

25.   What cost 7 cords of wood, at 4 dollars a cord?

26.   What cost 10 skeins of silk, at 4 cents a skein?

27.   If you can buy 4 nuts for 1 cent, how many nuts can you buy for 8 cents? For 9 cents? For 11 cents? For 12 cents?

28.   How many are 5 times 5? 5 times 6? 5 times 8? 5 times 9?

29.   At 5 cents apiece, what cost 7 lead-pencils? 10 lead-pencils? 11 lead-pencils? 12 lead-pencils?

30.   When 6 dollars are paid for a cord of wood, what must be paid for 5 cords? For 6 cords? For 7 cords?

31.   How many are 6 times 8? 6 times 9? 6 times 10? 6 times 11? 6 times 12?

32.   If a horse travels 7 miles in 1 hour, how far will he travel in 7 hours? In 9 hours? In 11 hours? In 12 hours?

33.   At 7 cents a pound, what will 8 pounds of beef cost? 10 pounds? 6 pounds? 12 pounds?

34.   When flour is 8 dollars a barrel, how much

must be paid for 5 barrels? 9 barrels? 8 barrels 10 barrels? 11 barrels? 12 barrels?

35. At 9 cents a pound, what cost 6 pounds of coffee? 9 pounds? 11 pounds? 12 pounds?

36. What cost 10 yards of cambric, at 12 cents a yard?

37. William sold 10 doves, at 10 cents apiece; how much did he receive for them?

38. What will 11 dozen of eggs cost, at 10 cents a dozen? At 11 cents? At 12 cents?

39. If 4 men can do a piece of work in 8 days, how long will it take one man to do it?

ANALYSIS. FULL FORM. — *It will take* 1 *man* 4 *times as long to do a piece of work as it takes* 4 *men; then, if it take* 4 *men* 8 *days to do it, it will take* ·1 *man* 4 *times* 8 *days, which are* 32 *days. Therefore, if* 4 *men can do a piece of work in* 8 *days, it will take* 1 *man* 32 *days to do it.*

ABBREVIATED FORM. — *Since it takes* 4 *men* 8 *days to do a piece of work, it will take* 1 *man* 4 *times* 8 *days, which are* 32 *days.*

40. If 4 men can do a piece of work in 8 days, how many men will it take to do it in one day?

ANALYSIS. FULL FORM. — *It will take* 8 *times as many men to do a piece of work in* 1 *day as to do it in* 8 *days; then, if it take* 4 *men to do it in* 8 *days, it will take* 8 *times* 4 *men to do it in* 1 *day;* 8 *times* 4 *men are* 32 *men. Therefore, if* 4 *men can do a piece of work in* 8 *days, it will take* 32 *men to do it in* 1 *day.*

ABBREVIATED FORM. — *Since it takes* 4 *men to do a piece of work in* 8 *days, it will take* 8 *times* 4 *men, or* 32 *men, to do it in* 1 *day.*

41. If 5 men can reap a field in 7 days, how long will it take one man?

42. If 10 men can reap a field in 9 days, how many men will it take to reap it in one day?

43. If a quantity of provisions will supply 11 men 8 days, how long will it supply one man?

44. If 6 pipes of equal size can together fill. a cistern in 12 hours, in how many hours could one of them fill it?

45. How many men must be employed to do a piece of work in one day, when it takes 11 men 12 days to do it?

46. If 12 men can dig a ditch in 12 days, how long will it take one man?

LESSON XII.

1. Four times three are how many?

   Ans.   *Four times three are twelve.*

2. Three times two are how many?

3. Seven times three are how many?

4. Four times four are how many?

5. Five times two are how many?

6. Three times seven are how many?

7. Five times four are how many?

8. Two times six are how many?

9. Six times five are how many?

10. Five times three are how many?

11. Six times four are how many?

12. Five times six are how many?

13. Eight times six are how many?

14. Seven times five are how many?

15. Six times six are how many?

16. Seven times four are how many?

17. Six times three are how many?

18. Seven times seven are how many?

19. Eight times five are how many?

20. Eight times four are how many?

21. Nine times three are how many?

22. Two times ten are how many?

23. Three times eight are how many?

24. Four times nine are how many?
25. Five times seven are how many?
26. Six times two are how many?
27. Seven times eight are how many?
28. Ten times three are how many?
29. Ten times six are how many?
30. Nine times four are how many?
31. Nine times five are how many?
32. Seven times naught are how many?
33. Three times three are how many?
34. Eleven times two are how many?
35. Twelve times one are how many?
36. Ten times seven are how many?
37. Nine times six are how many?
38. Twelve times two are how many?
39. Eleven times four are how many?
40. Ten times eight are how many?
41. Twelve times five are how many?
42. Nine times eight are how many?
43. Eight times eight are how many?
44. Seven times six are how many?
45. Eleven times three are how many?
46. Eleven times five are how many?
47. Ten times nine are how many?
48. Eleven times six are how many?
49. Twelve times four are how many?
50. Eleven times seven are how many?
51. Eleven times eight are how many?
52. Twelve times six are how many?
53. Twelve times seven are how many?
54. Twelve times nine are how many?
55. Eleven times nine are how many?
56. Twelve times eight are how many?
57. Eleven times twelve are how many?
58. Eleven times eleven are how many?
59. Twelve times ten are how many?
60. Twelve times twelve are how many?

LESSON XIII.

1.   At 9 dollars apiece, what will 8 plows cost?

2.   If 5 men can reap a field in 10 days, how long will it take one man to reap it?

3.   James bought 3 oranges, at 5 cents each, and 6 lemons, at 2 cents each; how much did the whole cost?

4.   Susan bought 8 yards of cotton cloth, at 9 cents a yard, and 4 skeins of thread, at 2 cents a skein; what was the cost of the whole?

5.   A lady bought 8 pounds of sugar, at 11 cents a pound, and paid 8 dozen of eggs, at 10 cents a dozen, and the remainder in money; how much money did she pay?

6.   What cost 7 quarts of cherries, at 6 cents a quart?

7.   How much more will 7 quarts of currants, at 7 cents a quart, cost, than 8 quarts of berries, at 6 cents a quart?

8.   How much more is 8 times 8 than 7 times 9?

9.   How much more is 8 times 9 than 10 times 7?   Than 6 times 12?

10.   If two men start from the same place, and travel in opposite directions, the one traveling at the rate of 3 miles an hour, and the other at the rate of 4 miles an hour, how far apart will they be at the end of 5 hours?

11.   If one man can make 6 pairs of shoes in one day, how many pairs can 9 men make?

12.   What cost 2 tons of hay, at 15 dollars a ton?

13.   Bought 6 cords of wood, at 8 dollars a cord, and handed in payment 5 ten-dollar bills; how much change should be received back?

14.   What cost 3 barrels of beef, at 16 dollars a barrel?

15. If one quire of paper costs 20 cents, what will 3 quires cost?   4 quires?   5 quires?

16. At 17 cents a yard, what will 2 yards of calico cost?   3 yards?

The SIGN of MULTIPLICATION is an inclined cross, $\times$, which signifies *times* or *multiplied by;* thus, $5 \times 4 = 20$ is read, 5 times 4, or 5 multiplied by 4, are equal to 20.

17. How many are 9×4? 10×8?

18. How much more is 10×10 than 11×9?

19. Two men start 50 miles apart, and travel towards each other, the one at the rate of 4 miles an hour, and the other at the rate of 3 miles; how far apart will they be at the end of 5 hours?

20. In a certain orchard there are 10 rows of trees, with 11 trees in each row; how many trees are there in the orchard?

21. A farmer sold 9 sheep, at 5 dollars apiece, and 5 lambs, at 3 dollars apiece; how much did he get for them all?

22. A tailor has a piece of broadcloth containing 33 yards; if he should cut from it 13 yards, what will the remainder be worth, at 4 dollars a yard?

23. Bought 6 writing-books, at 8 cents apiece, and 5 more, at 6 cents apiece, and sold the whole for 90 cents; how much was made by the sale?

24. A farmer has his wheat in 5 bins, containing 10 bushels each; how much is the whole worth, at 2 dollars a bushel?

25. Five times twelve, less ten, plus fifteen, are how many?

26. Four times fifteen, less twenty, plus six, plus twelve, are how many?

27. Five times twenty, less twenty-five, are how many?

28. Eight times eleven, plus twelve, less thirty, are how many?

29. Ten times eight, plus eighteen, less twenty, less six, are how many ?

30. Four times sixteen, plus six, less four times five, are how many ?

31. Five times twenty-five, less fifty, plus eight, are how many ?

32. Eight times twenty-five, less five times ten, are how many ?

33. If a man earns 50 dollars in 5 weeks, and pays of his earnings 3 dollars a week for board, how much will he have left ?

34. If one plow is worth 3 cords of wood, how many cords will 15 plows cost ?

35. If a man earns 100 cents a day, and pays out for family expenses 60 cents, how much will he have left at the end of 5 days ?

36. By putting in the savings bank 14 dollars a month, how much may be saved in 6 months ?

37. For how much must I sell 6 cows, which cost 25 dollars each, to gain 25 dollars ?

38. A boy earns 12 cents every day, and spends 4 cents ; how much money will he have at the end of 12 days ?

39. How many are 5 times 16, plus 20 ?   4 times 25, minus 30 ?

40. How many are $20 \times 6 + 15$? $21 \times 6 - 11$? $17 \times 4 - 16$?

41. George has 15 marbles, and Lewis has 3 times as many, less 10 ; how many has Lewis ?

42. Laura gathered 4 quarts of strawberries, and Mary gathered 3 times as many, less 2 quarts ; how many did Mary gather ?

43. What cost 18 pounds of flour, at 5 cents a pound ?

44. What cost 4 pecks of potatoes, at 19 cents a peck ?

45. A person poured into a can 16 quarts of oil at 4 different times, and from the same can filled 3 jugs, holding 8 quarts each; how much oil remained in the can?

46. If John has 3 chickens and Joseph 5 times as many, how many has Henry, who has as many as John and Joseph both, plus 4?

47. Three boys had some nuts given them, Robert receiving 20, William 2 times as many as Robert, lacking 15, and Ezra as many as Robert and William both; how many did Ezra receive?

48. Alfred and Edward sold the same quantity of berries; Alfred received 4 ten-cent pieces and 4 three-cent pieces; Edward received a fifty-cent piece and a five-cent piece, and paid back in making change a three-cent piece; how much more money did the one obtain than the other?

LESSON XIV.

DIVISION is the process of finding how many times one number is contained in another, or the process of separating a number into any proposed number of equal parts. The result obtained is called the *quotient*. The excess that is sometimes left after dividing is called the *remainder*.

1. At 2 cents each, how many peaches can be bought for 4 cents?

ANALYSIS. — *Since 1 peach can be bought for 2 cents, as many peaches can be bought for 4 cents as 2 cents are contained times in 4 cents, which are 2. Therefore, at 2 cents each, 2 peaches can be bought for 4 cents.*

2. At 3 cents each, how many lemons can be bought for 6 cents?

8. How many oranges, at 4 cents apiece, can be bought for 8 cents?

4. How many pounds of sugar, at 8 cents a pound, can be bought for 24 cents?

5. Lucy paid 20 cents for milk, at 5 cents per quart; how many quarts did she buy?

6. Harry divided 9 apples among his sisters, giving 3 to each; how many sisters had he?

7. If I walk 4 miles an hour, how long will it take me to walk 36 miles?

8. John writes 6 lines a day; in how many days will he write 18 lines?

9. If a scholar explains 2 examples in one minute, in how many minutes will he explain 12 examples?

10. At 5 shillings a bushel, how many bushels of apples can be bought for 15 shillings?

11. At 2 dollars a barrel, how many barrels of apples can be bought for 14 dollars?

12. If there are 7 days in one week, how many weeks are there in 56 days?

13. How many skeins of silk, at 3 cents a skein, can be bought for 21 cents?

14. How many yards of broadcloth, at 5 dollars a yard, can be bought for 25 dollars?

15. If 6 nuts cost 1 cent, how much will 24 cost?

16. At 9 dollars a barrel, how many barrels of flour can be bought for 18 dollars?

17. At 7 cents a pound, how many pounds of rice can be bought for 28 cents?

18. If 1 bag contains 3 bushels, how many such bags will be required to contain 36 bushels?

19. If a horse trot 6 miles an hour, how long will it take him to trot 30 miles?

20. How many pears, at 4 cents apiece, can you purchase for 16 cents?

21. In how many hours will a ship sail 81 miles, at the rate of 9 miles an hour?

22. At 6 dimes a bushel, how many bushels of corn can be bought for 36 dimes?

23. How many boxes of strawberries, at 3 dimes apiece, can be bought for 33 dimes?

24. How many yards of cloth, at 4 dollars a yard, can you buy for 40 dollars?

25. At 9 cents a quart, how many quarts of molasses can you purchase for 45 cents?

26. At 8 cents a paper, how many papers of pins may be bought for 32 cents?

27. If I give 8 cherries for one apple, how many apples shall I receive for 48 cherries?

28. If you have 30 cents, how many pencils can you buy, at 5 cents apiece?

29. If it take 11 yards to make one dress, how many dresses can be made from 44 yards?

30. When coal is 10 dollars a ton, how many tons can be bought for 60 dollars?

31. If 9 cherries cost one cent, how much will 72 cherries cost?

32. At 7 shillings a day, how many days will it take a man to earn 49 shillings?

33. For 63 dollars how many plows can be bought, at 9 dollars each?

34. At 8 dimes a day, how long will a man be in earning 64 dimes?

35. How much will 60 yards of cloth cost, at the rate of 5 yards for a dollar?

36. At 12 dollars a week, how long will it require to earn 72 dollars?

37. For 40 apples how many melons can be purchased, at the rate of 8 apples for one melon?

38. At 9 dimes each, how many turkeys can be purchased for 108 dimes?

39. If a train of cars moves at the rate of 12 miles an hour, how many hours will it require to move 96 miles?

40. For 24 cents how many oranges can be bought, at 3 cents apiece? At 4 cents apiece? At 6 cents apiece?

41. For 36 cents how many pounds of sugar can be bought, at 9 cents a pound? At 12 cents a pound?

42. A school consisting of 120 pupils is divided into a certain number of classes, each class containing 12 pupils; how many classes are there?

43. For 51 dollars how many acres of land can be bought, at the rate of 17 dollars an acre?

44. If one man can do a piece of work in 27 days, how many men will be required to do the same work in 3 days?

45. In how many days will 5 men reap a field, if 40 men can reap it in one day?

46. If one pipe can fill a cistern in 144 hours, how many such pipes will be required to fill it in 12 hours?

LESSON XV.

1. Nine are how many times 3?

2. Eleven are how many times 4?

 ANS. 11 *are 2 times 4, with a remainder of* 3.

3. Twelve are how many times 2? 3? 4?

4. Thirteen are how many times 2? 4? 5?

5. Fourteen are how many times 2? 7?

6. Fifteen are how many times 3? 5? 6?

7. Sixteen are how many times 2? 4? 8?

8. Seventeen are how many times 2? 3? 5?

9. Eighteen are how many times 2? 3? 6? 9? 8?

10. Nineteen are how many times 9? 8? 7? 6? 5?

11. Twenty are how many times 2? 4? 5? 10? 11?

12. Twenty-one are how many times 3? 7? 10? 12?

13. Twenty-two are how many times 2? 11?

14. Twenty-three are how many times 2? 10? 11? 9?

15. Twenty-four are how many times 4? 6? 8? 12?

16. Twenty-five are how many times 5? 6? 10? 12?

17. Twenty-six are how many times 2? 6? 13?

18. Twenty-eight are how many times 2? 4? 7? 14?

19. Thirty are how many times 2? 3? 5? 6? 10? 15?

20. Thirty-one are how many times 3? 6? 9?

21. Thirty-two are how many times 4? 8? 16?

22. Thirty-four are how many times 6? 8? 10? 17?

23. Thirty-five are how many times 5? 7? 8? 10? 11?

24. Thirty-six are how many times 4? 6? 9? 12? 18?

25. Thirty-seven are how many times 8? 9? 15? 16?

26. Forty are how many times 4? 8? 10? 15? 20?

27. Forty-two are how many times 6? 7? 10? 14? 21?

28. Forty-five are how many times 5? 9? 10? 11? 15?

29. Forty-eight are how many times 6? 8? 12? 16?

30. Fifty are how many times 5? 7? 9? 10?

31. Fifty-one are how many times 3? 5? 17?

32. Fifty-four are how many times 6? 9? 10? 12? 18?

33. Fifty-six are how many times 6? 7? 8? 9? 14?

34. Sixty are how many times 3? 4? 12? 15? 30?

35. Sixty-four are how many times 4? 8? 16? 12? 30?

36. Sixty-five are how many times 5? 8? 10? 12? 13?

37. Sixty-nine are how many times 6? 7? 20?

38. Seventy are how many times 5? 10? 12?

39. Seventy-two are how many times 8? 12?

40. Seventy-four are how many times 10? 12?

41. Seventy-five are how many times 3? 5? 15? 25?

42. Seventy-seven are how many times 7? 9? 10? 11?

43. Eighty are how many times 4? 5? 8? 10? 20? 40?

44. Eighty-four are how many times 4? 7? 12?

45. Eighty-five are how many times 5? 8? 12?

46. Eighty-eight are how many times 4? 8? 10? 11? 20?

47. Ninety are how many times 2? 3? 5? 9? 30?

48. Ninety-two are how many times 9? 10? 15? 20?

49. Ninety-five are how many times 5? 10? 11? 12?

50. Ninety-six are how many times 6? 8? 12?

51. Ninety-nine are how many times 3? 9? 10? 11?

52. One hundred are how many times 4? 5? 20? 25? 50?

53. One hundred and eight are how many times 9? 12? 25?

54. One hundred and ten are how many times 5? 10? 11? 20?

55. One hundred and twelve are how many times 2? 4? 25?

56. One hundred and twenty are how many times 2? 5? 8? 12? 15? 20?

57. One hundred and twenty-five are how many times 5? 20? 25? 50?

58. One hundred and fifty are how many times 10? 15? 25?

LESSON XVI.

1. Harry had 6 chestnuts, which he gave in equal numbers to 2 of his brothers; how many did each receive?

ANALYSIS. FULL FORM. — *One brother received one half as many as the 2 brothers; then, if the 2 brothers received 6 chestnuts, 1 brother received one half of 6 chestnuts, which is 3 chestnuts. Therefore, if 6 chestnuts were divided equally between 2 brothers, each of them received 3 chestnuts.*

ABBREVIATED FORM. — *Since the 2 brothers received 6 chestnuts, 1 brother received one half of 6 chestnuts, which is 3 chestnuts.*

NOTE. — When any number is divided into *two* equal parts, one of those parts is called *one half;* when into *three* equal parts, one of those parts is called *one third;* when into *four* equal parts, one of those parts is called *one fourth;* and so on.

2. Lucy divided 9 apples equally among her 3 sisters; how many did she give to each of them?

3. Thomas has 8 apples, which he wishes to divide equally between 2 boys; how many can he give to each of them?

4. When 4 cords of wood cost 20 dollars, how much does one cord cost ?

5. If you wish to give 8 oranges to 4 persons, how many can you give to each, if you divide them equally ?

6. Mary has 10 pins, in 2 cushions ; if equally divided, how many are there in each cushion ?

7. Thomas distributed 10 flowers equally among 5 of his playmates ; how many did each receive ?

8. If 12 dollars be divided equally among 6 men, how many will each receive ?

9. James has 12 peaches, which he wishes to give to 4 of his companions ; how many can he give to each, if he divides them equally ?

10. If a man pays 35 cents for 5 pounds of nails, how much are the nails a pound ?

11. Mary divided 42 apples equally among 7 companions ; how many did she give to each ?

12. In an orchard there are 56 trees, standing in 8 equal rows ; how many trees are there in each row ?

13. When 5 yards of cloth are bought for 20 dollars, what is the price a yard ?

14. If 5 dollars will buy 40 yards of cotton cloth, how many yards will one dollar buy ?

15. When 27 cherries cost 9 cents, how many cherries can be bought for one cent ?

16. If 3 oranges are worth as much as 12 apples, how many apples is one orange worth ?

17. If 9 yards of cloth cost 108 cents, how much will one yard cost ?

18. When 50 dollars are paid for 10 cords of wood, how much is it a cord ?

19. If there are 77 days in 11 weeks, how many days are there in one week ?

20. If 12 tons of coal cost 96 dollars, what does one ton cost ?

21.   What is one seventh of 49 apples ?

22.   What is one sixth of 66 oranges ?

23.   80 apples are 8 times what number of apples ?

24.   99 are 9 times what number ?

25.   One man can do a certain piece of work in 25 days; how long will it take 5 men to do it ?

26.   How many men will reap a field in 6 days, if 54 men can reap it in one day ?

27.   If one pipe can fill a cistern in 84 hours, in how long a time will 7 such pipes fill it ?

28.   How many tons of hay can be bought for 60 dollars, at 10 dollars a ton ? At 12 dollars a ton ? At 15 dollars a ton ?

29.   If two boats are 50 miles apart, and the one gains on the other 5 miles an hour, in how many hours will they be together ?

30.   A school, consisting of 120 pupils, is divided into 10 equal classes ; how many pupils are there in each class ?

31.   3 heifers were bought for 48 dollars ; how much were they apiece ?

32.   If a man earns 8 dollars a week, how long will it take him to earn 48 dollars ? 64 dollars ?

33.   How many cloaks, containing 9 yards, can be made from 63 yards of cloth ? From 72 yards ?

34.   If 51 dollars be divided among 3 men, how many dollars will each receive ?

35.   For 72 cents, how many pounds of beef can be bought, at 8 cents a pound ? At 12 cents ?

36.   If you should have 31 cents, how many writing-books could you buy, at 8 cents each, and how many cents would you have left ?

37.   If you had 57 dollars, how many sheep could you buy, at 5 dollars each, and how many dollars would you have left ?

38.   If 132 dollars will buy 12 coats, how much will one coat cost ?

39.   How many cords of wood can be bought for 100 dollars, at 4 dollars a cord ?

40.   When a man, having 75 dollars, can buy 9 pigs, and have 3 dollars left, what is the cost of each pig ?

41.   What is sugar a pound, if 9 pounds can be bought for 54 cents ?   For 63 cents ?   For 90 cents ?

42.   What is one twelfth of 84 dollars ?

43.   How many times 11 dollars in 121 dollars ?

44.   81 are 9 times what number ?

LESSON XVII.

1.   How many times $5 + 2$ in 21 ?   In 35 ?

2.   How many times $6 + 4$ in 50 ?   In 70 ?

3. $44 + 22$ are 11 times what number ?

4.   What is one eighth of $55 + 9$?   Of $63 + 9$?

5.   How many times 10 in $115 - 5$?   In $107 - 7$?

6.   What is one ninth of $66 - 3$?   Of $78 - 6$?

7.   How many times 9 less 4 in $63 - 8$?

8.   How many times $18 - 12$ in $90 - 30$?

9.   Jason had 52 apples, and found 8 more ; he then divided the whole equally among 4 schoolmates ; how many did he give to each ?

10.   A man had 25 cows, and bought 35 more ; if he should put them all into 5 pastures, how many would there be in each pasture, if equally divided ?

11.   A farmer had 47 bushels of apples ; saving 12 bushels for his own use, he sold the rest in equal quantities to 5 persons ; how many bushels did he sell to each person ?

12.   A boy on the way to market with 19 fishes, lost 4 of them ; the rest he sold for 75 cents ; how much apiece did he get for those he sold ?

13. William had 16 chickens, but a cat caught 4 of them; the rest he sold for 60 dimes; how much apiece did he get for those he sold?

14. Thomas sold some nuts for 12 cents, some apples for 18 cents, and some peaches for 20 cents, and with the money bought writing-books at 10 cents apiece; how many writing-books did he buy?

15. A gentleman divided 120 dollars equally among his 3 sons and 2 daughters; how many dollars did he give to each?

16. George had 59 pears; he gave 4 of them to one companion, 3 to another, 2 to another, and divided the remainder equally among his 10 classmates; how many did each receive?

17. Bought 6 barrels of pork for 120 dollars; at how much a barrel must it be sold to gain 2 dollars a barrel?

18. 2 men bought a cow for 40 dollars, and a horse for 80 dollars; they sold the cow for 30 dollars, and the horse for 100 dollars; what was each man's share of the gain, if equally divided?

19. How many pears, at 3 cents apiece, will pay for 2 melons, at 6 cents apiece?

ANALYSIS. — *Since* 1 *melon is worth* 6 *cents,* 2 *melons will be worth* 2 *times* 6 *cents, which are* 12 *cents; and since* 1 *pear is worth* 3 *cents, it will take as many pears to be worth* 12 *cents as* 3 *cents are contained times in* 12 *cents, which are* 4. *Therefore* 4 *pears, at* 3 *cents apiece, will pay for* 2 *melons, at* 6 *cents apiece.*

20. How many lemons, at 4 cents apiece, will pay for 4 oranges, at 2 cents apiece?

21. 3 times 8 are how many times 6? 4?

22. 3 times 10 are how many times 6? 5?

23. At 2 dollars a bushel, how many bushels of wheat must be given for 4 barrels of flour, at 8 dollars a barrel?

24. 4 times 7 are how many times 2 ?   14 ?

25. 4 times 9 are how many times 3 ?   12 ?

26. If a man can do a certain piece of work in 4 days when the days are 12 hours long, in how many days will he do it when the days are 8 hours long ?

27. 5 times 8 are how many times 4 ?   10 ?

28. 5 times 10 are how many times 2 ?   25 ?

29. How many yards of broadcloth, at 4 dollars a yard, should be received in payment for 10 sheep, at 6 dollars each ?

30. At 8 cents a pound, how many pounds of sugar can be bought for 6 dozen of eggs, at 12 cents a dozen ?

31. If you should sell 7 quarts of chestnuts, at 8 cents a quart, how many slates could you buy, at 14 cents apiece ?

32. If you should sell 8 quarts of milk, at 5 cents a quart, how many yards of cotton cloth, at 10 cents a yard, could you take in pay ?

33. 6 times 6 are how many times 3 ?   18 ?

34. 6 times 9 are how many times 2 ?   27 ?

35. 6 times 15 are how many times 3 times 10 ?

36. 7 times 10 are how many times 5 times 7 ?

37. 8 times 8 are how many times 4 times 4 ?

38. Bought 9 quarts of cranberries, at 10 cents a quart ; to pay for them gave raisins worth 15 cents a pound ; how many pounds did it take ?

39. Bought 9 yards of broadcloth at 8 dollars a yard, and paid for it with flour, at 12 dollars a barrel ; how many barrels did it take ?

The Sign of Division is a short horizontal line with a dot above it and another below, $\div$, which signifies *divided by;* thus, $18 \div 2 = 9$ is read, 18 divided by 2 is equal to 9.

40. 9 times 8 are how many times $24 \div 2$?

41. 9 times 11 are how many times $33 \div 3$?

42. 10 times 6 are how many times $20 \div 4$?

43. 5 times 10 are how many times 45 ÷ 9 ?

44. 4 times 12 are how many times 42 ÷ 7 ?

45. 12 times 7 are how many times 24 ÷ 6 ?

46. 3 times 8 are how many times 54 ÷ 9 ?

47. Gave 12 oranges in exchange for 6 pine-apples, worth 10 cents apiece ; how much was each orange worth ?

ANALYSIS. — *Since* 1 *pine-apple was worth* 10 *cents,* 6 *pine-apples would be worth* 6 *times* 10 *cents, which are* 60 *cents ; and since* 12 *oranges were worth* 60 *cents,* 1 *orange would be worth one twelfth of* 60 *cents, which is* 5 *cents ; Therefore, &c.*

48. Gave 6 cords of wood in exchange for 8 buffalo-robes, at 12 dollars each ; how much was the wood worth a cord ?

49. Received 25 yards of broadcloth for 5 tons of hay, worth 10 dollars a ton ; how much did I pay a yard for the cloth ?

50. One book contains 10 pages, with 30 lines on a page ; but a second book, containing the same number of lines, has 15 pages ; how many lines to a page has the second book ?

51. An orchard contains 8 rows of trees, and each row contains 12 trees ; if I should arrange the same number of trees in 6 equal rows, how many trees would each row contain ?

52. How much butter, at 20 cents a pound, must be given for 8 yards of calico, at 15 cents a yard ?

53. James bought 4 dozen of lead-pencils, at 25 cents a dozen, and paid for them in apples, at 10 cents a dozen ; how many dozen of apples did the pencils cost ?

54. Bought 15 yards of cloth, at 6 dollars a yard, and 4 yards more, at 5 dollars a yard, and paid for it with 11 loads of hay ; how much was the hay worth a load ?

55. A man can do a piece of work in 5 days, by working 8 hours a day; how many hours a day must he work to do it in 4 days?

56. If a certain amount of provisions will last 8 men 6 months, how long will it last 12 men?

57. If 9 men can build a wall in 5 days, how many men must be employed to build it in 3 days?

58. If it take 4 men 8 days to do a piece of work, how long will it take 16 men to do it?

59. How many men in 8 days can do a piece of work which will require 16 men 2 days to do?

60. If one pine-apple is worth as much as 2 oranges, and one orange is worth as much as 3 apples, how many apples are worth as much as 4 pine-apples?

ANALYSIS. — *If 1 pine-apple is worth 2 oranges, and 1 orange is worth 3 apples, then 2 oranges, or 1 pine-apple, will be worth 2 times 3 apples, which are 6 apples; and if 1 pine-apple is worth 6 apples, 4 pine-apples will be worth 4 times 6 apples, which are 24 apples.*

61. If one horse is worth as much as 3 cows, and one cow is worth as much as 8 sheep, how many sheep are worth as much as 2 horses?

62. If one pound of sugar cost 3 times as much as a pound of flour, and a pound of flour cost 5 cents, how many cents will 4 pounds of sugar cost?

LESSON XVIII.

1. If 4 barrels of apples cost 8 dollars, what cost 5 barrels?

ANALYSIS. — *If 4 barrels of apples cost 8 dollars, 1 barrel will cost one fourth of 8 dollars, which is 2 dollars; and if 1 barrel of apples cost 2 dollars, 5 barrels will cost 5 times 2 dollars, which are 10 dollars.*

2. If 3 pounds of butter cost 36 cents, what cost 5 pounds ?

3. When 10 cents are paid for 5 rolls of candy, how much must be paid for 6 rolls ?

4. When 5 dollars will buy 15 yards of cloth, how many yards will 9 dollars buy ?

5. If 4 tons of hay cost 40 dollars, what cost 3 tons ?

6. When 7 quarts of fruit bring 35 cents, how much do 5 quarts bring ?

7. If 24 brooms cost 6 dollars, how many brooms can be bought for 8 dollars ?

8. If 8 bushels of wheat cost 16 dollars, how much will 7 bushels cost ?

9. What will 9 quarts of milk cost, if 10 quarts cost 50 cents ?

10. If a horse trot 36 miles in 6 hours, how many miles will he trot in 11 hours ?

11. What will 12 yards of broadcloth cost, if 15 yards cost 75 dollars ?

12. If a man can earn 90 dollars in 9 weeks, how many dollars can he earn in 6 weeks ?

13. If 3 pounds of cheese cost 30 cents, what will 6 pounds cost ?

14. What cost 3 yards of cambric, if 6 yards cost 60 cents ?

15. What cost 17 weeks' board, if 9 weeks' board cost 27 dollars ?

16. What cost 4 tons of coal, at the rate of 10 tons for 70 dollars ?

17. When 56 cents are paid for 4 dozen of eggs, how much must be paid for 10 dozen ?

18. If 28 cents will buy 4 quarts of berries, how many quarts will 35 cents buy ?

ANALYSIS. — *If 4 quarts of berries cost 28 cents, 1 quart will cost one fourth of 28 cents, which is 7 cents ; and if*

cents will buy 1 quart of berries, 35 cents will buy as many quarts as 7 cents are contained times in 85 cents, which are 5. Therefore 5 quarts of berries can be bought for 35 cents, if 4 quarts cost 28 cents.

19. When 64 dollars are paid for 8 barrels of flour, how many barrels can be bought for 72 dollars?

20. How many pounds of veal can be bought for 96 cents, when 9 pounds cost 72 cents?

21. When 4 quarts of vinegar can be bought for 36 cents, how many quarts can be bought for 108 cents?

22. When 7 yards of cotton cloth can be bought for 84 cents, how many yards can be bought for 120 cents?

23. If 4 dollars will buy 32 yards of cotton cloth, how many dollars will buy 40 yards?

24. If 4 cents buy 8 apples, how much will 20 apples cost?

25. If 6 copies of a book cost 8 dollars, how much will 12 copies cost?

ANALYSIS. — *If 6 copies of a book cost 8 dollars, 12 copies will cost as many times 8 dollars as 6 copies are contained times in 12 copies, which are 2 ; therefore 12 copies will cost 2 times 8 dollars, which are 16 dollars.*

26. If 3 hats cost 7 dollars, how much will 9 hats cost?

27. When 6 cents will buy 5 apples, how many apples will 60 cents buy?

28. How many oranges can be bought for 63 cents, at the rate of 4 for 9 cents?

29. If 4 cherries are worth as much as one plum, and 3 plums are worth as much as one peach, how many peaches are worth as much as 24 cherries?

NOTE. — Find, as in Example 60, Lesson XVII., that 1 peach is worth 12 cherries ; then, if 12 cherries are worth 1 peach, 24 cherries will be worth as many peaches as 12 cherries are contained times in 24 cherries, which are 2.

30.   If 2 bushels of oats are worth 1 bushel of corn, and 2 bushels of corn are worth 1 bushel of wheat, how many bushels of wheat are worth 20 bushels of oats ?

31.   What will 19 pounds of chalk cost, if 13 pounds cost 65 cents ?

32.   When wheat is sold at the rate of 6 bushels for 12 dollars, how many bushels must be given for 4 cords of wood, at 5 dollars a cord ?

33.   If 8 dozen of eggs are sold for 96 cents, how many dozen will it take to buy 6 yards of gingham, at 18 cents a yard ?

34.   A steamboat can run 8 miles an hour down river, and only 6 miles an hour up river ; after running down river for 3 hours, how long will it be in returning ?

35.   A boat sailed up river for 10 hours, at the rate of 3 miles an hour, but returned over the same distance in 6 hours ; how fast did it sail down stream ?

36.   If 12 barrels of flour cost 84 dollars, what will 7 barrels cost ?   11 barrels ?   6 barrels ?   10 barrels ?   9 barrels ?

37.   A man bought 12 bushels of wheat, at 2 dollars a bushel, and 4 yards of cloth, at 3 dollars a yard, and paid for them in work, at 9 dollars a week ; how many weeks did he work ?

38.   If you should buy 60 pears, at the rate of 4 for 3 cents, and sell them at a cent apiece, how much would you make by the operation ?

39.   If 10 men can do a piece of work in 12 days, how many men must be employed to do it in 15 days?

40.   When 7 cords of wood can be bought for 56 dollars, how many cords of wood can be bought for 72 dollars ?

41.   If 9 men can dig a ditch in 8 days, how long will it take 12 men to dig it ?

3

42. How many books, at 6 dimes each, will
books, at 4 dimes each, pay for ?

43. A can travel at the rate of 5 miles an hour,
and B 7 miles ; they set out from the same point,
and in the same direction, but B starts after A has
traveled 30 miles ; how long will it take B to
overtake A ?

44. William can run 40 rods in 5 minutes, and
Jason the same distance in 4 minutes ; how long
will it take Jason to gain 30 rods on William ?

45. Three men buy a horse, A paying 12 dol-
lars, B paying twice as much as A, lacking 4 dollars,
and C paying 28 dollars ; what would have been the
cost to each person, if the expense had been shared
equally ?

46. If 3 cords of wood are worth 18 dollars,
and 10 cords are given for 12 thousand of shingles,
how much are the shingles a thousand ?

47. How much rye, at 9 dimes a bushel, must
be given for 27 bushels of buckwheat, at 3 dimes a
bushel ?

48. If 3 pounds of flour are worth 1 pound of
sugar, and 6 pounds of sugar are worth 1 pound of
tea, how many pounds of flour are worth 2 pounds
of tea ?

49. If 6 men can reap a field in 4 days, in how
many days can 8 men reap it ?

50. When 3 pounds of cheese cost as much as
1 pound of butter, and 3 pounds of butter as much
as 1 pound of tea, how many pounds of tea cost as
much as 81 pounds of cheese ?

51. If a cistern, capable of holding 60 gallons,
has a pipe by which 10 gallons can run into it in one
hour, and another pipe by which 5 gallons can run
out of it in the same time, when both pipes are
running, in what time will the cistern be filled ?

LESSON XIX.

TABLE OF UNITED STATES MONEY.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes, or 100 cents,	"	1 Dollar,	"	\$.
10 Dollars	"	1 Eagle,	"	E.

NOTE.— Dollars and cents written together are separated by a point (.); thus, \$ 4.50 is read 4 dollars 50 cents.

REDUCTION is the process of changing a number into one of a different denomination, but of equal value.

1. How many mills are there in 9 cents ?

ANALYSIS. ABBREVIATED FORM.— *Since in 1 cent there are 10 mills, in 9 cents there are 9 times 10 mills, which are 90 mills.*

2. How many cents are there in 80 mills ?

ANALYSIS.— *As there is 1 cent in 10 mills, there are as many cents in 80 mills as 10 mills are contained times in 80 mills, which are 8.*

3. How many cents in 1 dime ? In 3 dimes ? In 7 dimes ? In 11 dimes ? In 14 dimes ?

4. How many dimes in 10 cents ? In 20 cents ? In 43 cents ? In 90 cents ? In 100 cents ? In 110 cents ? In 150 cents ?

NOTE.— In 43 cents there are 4 dimes, and 3 cents remaining.

5. How many dimes in \$ 1 ? In \$ 3 ? In \$ 7 ?

6. How many dollars in 1 eagle ? In 2 eagles ? In 7 eagles ? In 11 eagles ?

7. How many eagles in 10 dollars ? In 20 dollars ? In 70 dollars ? In 120 dollars ?

8. How many cents in \$ 1 ? In \$ 2 ? In \$ 5 ? In \$ 9 ?

9. How many dollars in 100 cents ? In 200 cents ? In 700 cents ? In 900 cents ?

10. At 5 mills a yard, how many cents will 20 yards of tape cost ?

11. At 2 dimes a pound, how many dollars will 30 pounds of spice cost ?

12. When butter is 2 dimes a pound, how many pounds will $5 buy ?

13. If 7 pounds of beef cost 70 cents, how many pounds can be bought for 1 eagle ?

NOTE. — One dollar is equal to 6 New England shillings, to 8 New York shillings, and to 5 Canada shillings.

14. Bought 40 yards of cloth at 2 New York shillings a yard ; how many dollars did I pay ?

15. Bought 24 packages of envelopes in Boston at the rate of 2 packages for a shilling ; how many dollars did they cost ?

16. If you could buy 24 apples for a Canada shilling, how many could you buy for a dollar ? How many for a New England shilling ? How many for a New York shilling ?

TABLE OF ENGLISH MONEY.

4 Farthings (far. or qr.) make 1 Penny,   marked d.
12 Pence   "   1 Shilling,   "   s.
20 Shillings   "   1 Pound,   "   £.

NOTE. — A Sovereign is a gold coin whose value is one pound sterling.

1. How many farthings in 1 penny ? In 2 pence ? In 6 pence ? In 11 pence ? In 20 pence ?

2. How many pence in 4 farthings ? In 12 farthings ? In 36 farthings ? In 48 farthings ?

3. How many pence in 1 shilling ? In 4 shillings ? In 7 shillings ? In 11 shillings ?

4. How many shillings in 12 pence ? In 48 pence ? In 36 pence ? In 108 pence ?

5. How many shillings in £1 ? In £4 ? In £7 ? In £9 ? In £10 ?

6. How many pounds in 20 shillings ? In 40 shillings ? In 80 shillings ? In 120 shillings ?

7. How many pence in 5s. 6d. ? In 8s. 9d. ? In 10s. 3d. ? In 12s. 6d. ?

NOTE. — In 5 shillings there are 60 pence, and 6 pence added make 66 pence.

8. How many shillings in £2 6s. ? In £3 9s. ? In £6 10s. ? In £10 8s. ?

9. At 3d. a pound, how many shillings will 24 pounds of rice cost ?

10. At 5s. a yard, how many pounds will 16 yards of carpeting cost ?

LESSON XX.

TABLE OF TROY, OR MINT WEIGHT.

24 Grains (gr.)	make 1 Pennyweight, marked pwt. or dwt.
20 Pennyweights	" 1 Ounce, " oz.
12 Ounces	" 1 Pound, " lb.

NOTE. — This weight is used in weighing gold, silver, and jewels.

1. How many grains in 1 pennyweight ? In 2 pennyweights ? In 4 pennyweights ?

2. How many pennyweights in 24 grains ? In 48 grains ? In 96 grains ?

3. How many pennyweights in 1 ounce ? In 2 ounces ? In 5 ounces ?

4. How many ounces in 20 pennyweights ? In 40 pennyweights ? In 100 pennyweights ?

5. How many ounces in 1 pound ? In 2 pounds ? In 5 pounds ? In 9 pounds ? In 10 pounds ? In 12 pounds ?

6. How many pounds in 12 ounces ? In 36 ounces ? In 60 ounces ? In 144 ounces ?

7. In 3oz. 10pwt. how many pennyweights ?

8. In 6lb. 7oz. how many ounces ?

9. At 6 cents a pennyweight, what will 4oz. 10pwt. of silver cost?

10. At 9 dimes a pennyweight, what must be paid for 1oz. 5pwt. of gold?

TABLE OF AVOIRDUPOIS WEIGHT.

16 Drams (dr.)	make	1 Ounce,	marked	oz.
16 Ounces	"	1 Pound,	"	lb.
25 Pounds	"	1 Quarter,	"	qr.
4 Quarters, or 100 lbs.	"	1 Hundred-weight,	"	cwt.
20 Hundred-weight	"	1 Ton,	"	T.

NOTE. — This weight is used in weighing almost every kind of goods, and all' metals except gold and silver.

1. How many ounces in 1 pound? In 2 pounds? In 5 pounds? In 10 pounds? In 32 drams?

2. How many pounds in 16 ounces? In 32 ounces? In 80 ounces? In 96 ounces?

3. How many pounds in 1 quarter? In 3 quarters? In 5 quarters? In 8 quarters?

4. How many quarters in 50 pounds? In 75 pounds? In 100 pounds?

5. How many hundred-weight in 4 quarters? In 8 quarters? In 16 quarters? In 20 quarters?

6. How many hundred-weight in 2 tons? In 3 tons? In 5 tons? In 6 tons?

7. How many tons in 20 hundred-weight? In 40 hundred-weight? In 80 hundred-weight? In 60 hundred-weight?

8. What cost 4 hundred-weight of sugar, at 9 cents a pound? At 8 cents?

9. What cost 6 tons of bone-dust, at 2 dollars a hundred-weight?

10. How many pounds in 1cwt. 2qr. 13lb.?

11. How much will 2cwt. 1qr. of beef cost, at 10 cents a pound?

12. If 5 cwt. of guano cost 15 dollars, how much will 3 tons cost?

LESSON XXI.

TABLE OF LINEAR, OR LONG MEASURE.

12	Inches (in.)	make	1 Foot,	marked ft.
3	Feet	"	1 Yard,	" yd.
5½	Yards, or 16½ feet,	"	1 Rod or Pole,	" rd.
40	Rods	"	1 Furlong,	" fur.
8	Furlongs, or 320 rods,	"	1 Mile,	" m.
3	Miles	"	1 League,	" lea.
69⅙	Miles (nearly)	"	1 Degree,	" deg. or °.
360	Degrees	"	1 Circle of the Earth.	

NOTE. — This measure is used in measuring distances in any direction.

5½ and 16½ are read, five and one half and sixteen and one half. 69⅙ is read sixty-nine and one sixth.

1. How many inches in 1 foot ? In 5 feet ? In 7 feet ? In 10 feet ? In 12 feet ?

2. How many feet in 24 inches ? In 36 inches ?

3. How many feet in 2 yards ? In 11 yards ? In 15 yards ? In 20 yards ?

4. How many yards in 6 feet ? In 12 feet ? In 18 feet ? In 24 feet ?

5. How many rods in 1 furlong ? In 3 furlongs ? In 7 furlongs ? In 1 mile ?

6. How many furlongs in 40 rods ? In 120 rods ? In 160 rods ?

7. How many furlongs in 1 mile ? In 3 miles ? In 10 miles ? In 12 miles ?

8. How many miles in 16 furlongs ? In 24 furlongs ? In 40 furlongs ? In 96 furlongs ?

9. How many miles in 3 leagues ? In 8 leagues ? In 12 leagues ? In 15 leagues ?

10. How many leagues in 9 miles ? In 24 miles ? In 30 miles ? In 42 miles ?

11. How many miles in 1 degree ?

12. How many furlongs in 4 leagues ? In 6 leagues ?

13. If it take 5 minutes to travel 1 furlong, how long will it take to travel 1 mile ? 1 league ? 2 leagues ?

14. How many inches in 2yd. 2ft. 6in. ? In 1yd. 2ft. 5in. ?

TABLE OF CLOTH MEASURE.

2¼	Inches (in.)	make	1 Nail,	marked	na.
4	Nails	"	1 Quarter of a yard,	"	qr.
4	Quarters	"	1 Yard,	"	yd.
3	Quarters	"	1 Ell Flemish,	"	E. F.
5	Quarters	"	1 Ell English,	"	E. E.

NOTE. — This measure is used in measuring cloth, and other articles sold by the yard or ell.

2¼ is read two and one fourth.

1. How many nails in 2 quarters ? In 3 quarters ? In 5 quarters ?

2. How many quarters in 12 nails ? In 16 nails ? In 20 nails ? In 24 nails ?

3. How many quarters in 1 yard ? In 4 yards ? In 12 yards ? In 20 yards ?

4. How many yards in 4 quarters ? In 20 quarters ? In 80 quarters ? In 100 quarters ?

5. How many quarters in 4 ells English ? In 8 ells English ? In 12 ells English ?

6. How many ells English in 20 quarters ? In 60 quarters ? In 80 quarters ?

7. What cost 10 yards of velvet, at $2 a quarter ? At $10 an ell English ?

8. How many nails in 3yd. 3qr. 1na. ?

9. How many quarters in 5yd. 3qr. ?

10. 23 quarters are equal to how many yards ?

11. At 3 cents a nail, how much will 3 ells English of cloth cost ?

12. If 5 nails of cloth cost 25 cents, what cost 5 yards ?

LESSON XXII.

TABLE OF SURFACE, OR SQUARE MEASURE.

144	Square inches (sq. in.) make	1	Square foot,	marked sq. ft.
9	Square feet "	1	Square yard,	" sq. yd.
30¼	Square yards "	1	Square rod or pole,	P.
40	Square rods "	1	Rood,	" R.
4	Roods "	1	Acre,	" A.
640	Acres "	1	Square mile,	" sq. m.

NOTE. — This measure is used in measuring surfaces of all kinds.

1. How many square feet in 1 square yard? In 3 square yards? In 7 square yards? In 8 square yards? In 9 square yards?

2. How many square yards in 9 square feet? In 36 square feet? In 81 square feet?

3. How many square rods in 1 rood? In 3 roods? In 7 roods?

4. How many roods in 80 square rods? In 120 square rods? In 160 square rods?

5. How many roods in 2 acres? In 6 acres? In 10 acres? In 12 acres?

6. How many acres in 8 roods? In 24 roods? In 60 roods? In 50 roods?

7. How many acres in 1 square mile?

8. How many square rods in a field 12 rods long and 9 rods wide?

9. At 5 dollars for 1 square rod, what cost 1 acre of land?

10. In 1A. 2R. 20P. are how many square rods?

TABLE OF CUBIC, OR SOLID MEASURE.

1728	Cubic inches (cu. in.) make	1	Cubic foot,	marked cu. ft.
27	" feet "	1	" yard,	" cu. yd.
40	" feet "	1	Ton,	" T.
16	" feet "	1	Cord foot,	" c. ft.
8	Cord feet, or }	1	Cord of wood,	" C.
128	Cubic feet, }			

3 *

NOTE. — Cubic measure is used in measuring such things as have length, breadth, and thickness; as timber, stone, &c.

1.   How many cubic feet in 1 cubic yard?   In 3 cubic yards?   In 4 cubic yards?

2.   How many cubic yards in 27 cubic feet?   In 54 cubic feet?   In 81 cubic feet?

3.   How many cubic feet in 2 tons of timber? In 3 tons of timber?   In 5 tons of timber?

4.   How many tons in 80 cubic feet of timber? In 120 cubic feet?   In 160 cubic feet?

5.   How many cord feet in 2 cords of wood? In 5 cords?   In 9 cords?

6.   How many cords in 16 cord feet?   In 24 cord feet?   In 80 cord feet?

7.   How many cubic feet in 1 cord of wood?

8.   What cost 5 cords of wood, if 4 cord feet cost $ 3?

9.   What cost 2 tons 20 cubic feet of timber, at $ 1 for 4 cubic feet?

LESSON XXIII.

LIQUID, OR WINE MEASURE.

4 Gills (gi.)	make	1 Pint,	marked	pt.
2 Pints	"	1 Quart,	"	qt.
4 Quarts	"	1 Gallon,	"	gal.
63 Gallons	"	1 Hogshead,	"	hhd.
2 Hogsheads	"	1 Pipe,	"	pi.
2 Pipes	"	1 Tun,	"	tun.

NOTE. — This measure is used in measuring all kinds of liquid, except, in some places, beer, ale, porter, and milk.

1.   How many gills in 4 pints?   In 8 pints?   In 12 pints?   In 15 pints?

2.   How many pints in 8 gills?   In 16 gills?   In 32 gills?   In 36 gills?

3. How many pints in 4 quarts? In 6 quarts? In 9 quarts? In 12 quarts?

4. How many quarts in 3 gallons? In 8 gallons? In 12 gallons? In 15 gallons?

5. How many gallons in 1 pipe?

6. How many hogsheads in 2 tuns? In 6 tuns?

7. How many tuns in 8 hogsheads? In 16 hogsheads? In 20 hogsheads?

8. What cost 8 gallons of vinegar, at 5 cents a quart? At 10 cents?

9. If 2 quarts of oil cost 48 cents, what cost 1 quart? 1 pint? 1 gill?

10. If 2 gills of molasses cost 4 cents, how much will 1 gallon cost?

11. If 3 quarts of oil cost 60 cents, what cost 3 gallons and 3 quarts?

12. How many gills in 2 quarts and 1 pint?

13. When 5 gallons of burning-fluid can be bought for 3 dollars, what cost 1 hogshead and 7 gallons?

TABLE OF DRY MEASURE.

2 Pints (pt.)	make	1 Quart,	marked	qt.
8 Quarts	"	1 Peck,	"	pk.
4 Pecks	"	1 Bushel,	"	bu.

NOTE. — This measure is used in measuring grain, fruit, salt, coal, &c.

1. How many pints in 4 quarts? In 6 quarts? In 10 quarts? In 11 quarts?

2. How many quarts in 12 pints? In 18 pints? In 20 pints? In 30 pints?

3. How many quarts in 4 pecks? In 8 pecks?

4. How many pecks in 16 quarts? In 32 quarts? In 64 quarts? In 40 quarts?

5. How many pecks in 4 bushels? In 5 bushels? In 9 bushels? In 6 bushels?

6.   How many bushels in 12 pecks?   In 32 pecks?
In 48 pecks?   In 50 pecks?

7.   What costs 1 bushel of corn, at 3 cents a
quart?   At 2 cents?

8.   What cost 2 pecks of cherries, at 4 cents a
pint?   At 5 cents?

9.   How many quarts in 2 bushels?   In 3 bush-
els?   In 10 bushels?

10.   How many quarts in 2bu. 3pk. 4qt.?

11.   If 3 pints of cherries cost 15 cents, what
will 1 bushel cost?

LESSON XXIV.

TABLE OF TIME.

60 Seconds (sec.)	make	1 Minute,	marked	m.
60 Minutes	"	1 Hour,	"	h.
24 Hours	"	1 Day,	"	d.
7 Days	"	1 Week,	"	w.
365¼ Days, or 52 } weeks, 1¼ days, }	"	1 Julian Year,	"	y.
12 Calendar months (mo.)	"	1 Year,		
100 Years	"	1 Century.		

NOTE. — This measure is applied to the various divisions and
subdivisions into which time is divided.

A common Year is one of 365 days, and a Leap Year is one
of 366 days.

The following table will exhibit the names of the months, and
the number of days in each.

Winter.	{	1st month,	January,	has 31 days.		
		2d "	February,	" 28 "	in Leap year 29.	
Spring.	{	3d "	March,	" 31 "		
		4th "	April,	" 30 "		
		5th "	May,	" 31 "		
Summer.	{	6th "	June,	" 30 "		
		7th "	July,	" 31 "		
		8th "	August,	" 31 "		

$$\left\{\begin{array}{l} \text{9th month, September, has 30 days.} \\ \text{10th \quad " \quad October, \quad " \quad 31 \quad "} \\ \text{11th \quad " \quad November, \quad " \quad 30 \quad "} \end{array}\right.$$

Autumn.

Winter.   12th   "   December,   "   31   "

The number of days in each month may be readily remembered by the following lines : —

> " Thirty days hath September,
> April, June, and November;
> And all the rest have thirty-one,
> Save February, which alone
> Hath twenty-eight; and this, in fine,
> One year in four hath twenty-nine."

1. How many seconds in 2 minutes ? In 3 minutes ? In 4 minutes ?

2. How many minutes in 120 seconds ?

3. How many minutes in 2 hours ? In 4 hours ?

4. How many hours in 2 days ? In 3 days ?

5. How many days in 48 hours ? In 72 hours ?

6. How many days in 4 weeks ? In 6 weeks ? In 9 weeks ? In 10 weeks ?

7. How many weeks in 14 days ? In 56 days ? In 63 days ? In 84 days ?

8. How many months in 6 years ? In 10 years ?

9. If you can read 6 pages in 12 minutes, how many hours will it take you to read 10 times as many pages ?

10. Charles is 8 years 3 months old, and John is 7 years 10 months; how many months older is Charles than John ?

11. If a man can earn 60 dollars in 3 months, in how many months can he earn 100 dollars ?

12. If in 5 hours 3 pairs of shoes can be made, in how many days, of 10 hours each, can 24 pairs be made ?

13. If a ship sails 5 miles an hour, how far will it sail in a week ?

14. If a man can earn 1 cent a minute, how much can he earn in a day, when he labors 8 hours a day ?

Miscellaneous Table.

12 Units	make	1 Dozen.
12 Dozen	"	1 Gross.
12 Gross	"	1 Great Gross.
20 Units	"	1 Score.
24 Sheets of paper	"	1 Quire.
20 Quires	"	1 Ream.
56 Pounds	"	1 Bushel of Corn.
60 Pounds	"	1 Bushel of Wheat.
196 Pounds	"	1 Barrel of Flour.
200 Pounds	"	1 Barrel of Beef or Pork.

1. How many units in 4 dozen ?   In 8 dozen ?

2. What cost 4 dozen peaches, at 2 cents apiece ?

3. What cost 1 gross of writing-books, at 10 cents each ?

4. How many score in 60?   In 80?   In 100?

5. What cost 12 score pounds of pork, at 10 cents a pound ?

6. What cost 1 gross of pens, at 5 cents a dozen ?

7. What costs 1 ream of paper, at 10 cents a quire?   At 12 cents?

8. What costs 1 quire of paper, when 3 sheets can be bought for 2 cents ?

9. At $11 a hundred-weight, what costs 1 barrel of beef?   3 barrels ?

10. Bought a barrel of pork at 12 cents a pound, and sold it at $15 a hundred-weight; how much was made by the sale ?

11. Bought wheat at 3 cents a pound, and sold it at $2 a bushel; how much was made on a bushel ?

12. Bought beef at $18 a barrel, and sold it at 12 cents a pound; how much was made on a barrel ?

13. What costs 1 ream of paper, at 2 sheets for a cent ?

14. How many score in 5 dozen ?   In 10 dozen ? In 15 dozen ?

LESSON XXV.

A FRACTION is an expression denoting one or more equal parts of a unit.

1.   When any thing or number is divided into two equal parts, what is each one of those parts called ?
  Ans.   *One half.*

2.   How many halves in a whole one ?
3.   What is meant by a half of anything ?
4.   How many halves are there in 2 ?

ANALYSIS. — *Since in* 1 *there are* 2 *halves, in* 2 *there are* 2 *times* 2 *halves, which are* 4 *halves.*

5   How many halves are there in 4 ?   In 5 ?   In 10 ?   In 30 ?   In 50 ?

6.   How many halves in 2 and 1 half?
  Ans.   5 *halves.*

7.   How many halves in 3 ?   In 4 and 1 half?   In 9 and 1 half?   In 11 and 1 half?   In 12 and 1 half?

8.   How many whole ones are there in 2 halves ?   In 6 halves ?

ANALYSIS. — *Since there is* 1 *whole one in* 2 *halves, there are as many whole ones in* 6 *halves as* 2 *halves are contained times in* 6 *halves, which are* 3.

9.   How many whole ones are there in 8 halves ?   In 24 halves ?   In 30 halves ?   In 50 halves ?

10.   How many whole ones in 21 halves ?
  Ans.   10 *whole ones, and one half remaining.*

11.   How many whole ones in 60 halves ?   In 61 halves ?   In 45 halves ?   In 27 halves ?

12.   When any thing or number is divided into three equal parts, what is each one of those parts called ?
  Ans.   *One third.*

13.   What are two of those equal parts called ?
  Ans.   *Two thirds.*

14.   How many thirds in a whole one ?

15. What is meant by one third of anything?
By two thirds?

16. How many thirds are there in 2? In 3?
In 6? In 9? In 10?

17. How many thirds in 2 and 1 third? In 4
and 2 thirds? In 11 and 1 third?

18. How many whole ones in 6 thirds? In 7
thirds? In 19 thirds? In 21 thirds? In 23 thirds?

19. If an apple be divided into 4 equal parts,
what is one of those parts called?

ANS. *One fourth, or one quarter.*

20. What are two of those parts called? What
are three of those parts called?

21. How many fourths in a whole one?

22. What is meant by a fourth of anything?
By two fourths? By three fourths?

23. How many fourths in 2? In 3 and 1 fourth?
In 5? In 6 and 3 fourths? - In 8?

24. How many whole ones in 8 fourths? In 9
fourths? In 12 fourths? In 18 fourths? In 20
fourths? In 31 fourths?

25. What is meant by 1 fifth of anything? By
2 fifths? By 3 fifths? By 4 fifths?

26. How many fifths in a whole one? In 5 whole
ones? In 8 whole ones?

27. How many sixths in a whole one? In 3?
In 2? In 10? In 11?

28. What is meant by 1 sixth of anything? By
2 sixths? By 5 sixths?

29. How many whole ones in 18 sixths? In 17
sixths? In 30 sixths? In 37 sixths? In 73 sixths?

30. How many sevenths in a whole one?

31. What is meant by 1 seventh of anything?
By 2 sevenths? By 4 sevenths? By 6 sevenths?

32. How many whole ones in 14 sevenths? In
17 sevenths? In 21 sevenths? In 39 sevenths?
In 68 sevenths? In 84 sevenths?

33. What is meant by 1 eighth of anything? By 3 eighths? By 5 eighths?

34. How many ninths in a whole one? How many tenths? How many elevenths?

35. In 5 whole ones how many tenths? How many elevenths? How many twelfths?

36. How many whole ones in 24 eighths? In 24 twelfths? In 72 ninths? In 53 tenths?

37. What is meant by 1 twenty-second?

38. If anything is divided into 31 equal parts, what is each one of those parts called?

39. When an orange is worth 4 cents, what is 1 half of it worth?

ANALYSIS. — *Since* 1 *orange is worth* 4 *cents,* 1 *half of it is worth* 1 *half of* 4 *cents, which is* 2 *cents.*

40. If a pine-apple is worth 9 cents, what is 1 third of it worth?

41. If a bushel of apples is worth 50 cents, what is 1 fifth of a bushel worth?

42. When 8 dollars are paid for a ton of coal, what must be paid for 1 fourth of a ton?

43. What must be paid for 1 tenth of a vessel worth 60 eagles?

44. When a cord of wood costs $6, what does 1 third of a cord cost? 2 thirds of a cord?

ANALYSIS. — *Since* 1 *cord costs* $6, 1 *third of a cord costs* 1 *third of* $6, *which is* $2; *and since* 1 *third of a cord costs* $2, 2 *thirds of a cord cost* 2 *times* $2, *or* $4.

45. If a barrel of sugar is worth $18, what is 1 third of a barrel worth? 2 thirds of a barrel?

46. If a bushel of corn costs 60 cents, what will 1 fourth of a bushel cost? 2 fourths? 3 fourths?

47. If a man can earn $42 in a month, how many dollars can he earn in 1 seventh of a month? In 2 sevenths? In 4 sevenths? In 6 sevenths?

48.   Harriet had 40 pins, and gave Maria 2 fifths of them ; how many did she give her ?

ANALYSIS.   1 *fifth of* 40 *pins is* 8 *pins, and* 2 *fifths of* 40 *pins are* 2 *times* 8 *pins, or* 16 *pins.   Therefore* 2 *fifths of* 40 *pins are* 16 *pins.*

49.   What is 5 sixths of 30 ?   Of 72 ?
50.   What is 7 eighths of 56 ?   Of 80 ?
51.   What is 5 twelfths of 60 ?   Of 84 ?
52.   If 1 half of an orange is worth 2 cents, what is the whole orange worth ?

ANALYSIS. — *If* 1 *half of an orange is worth* 2 *cents,* 2 *halves, or the whole orange, is worth* 2 *times* 2 *cents, or* 4 *cents.*

53.   If 1 third of an orange is worth 2 cents, what is the whole of it worth ?
54.   If 1 fourth of a pound of beef is worth 3 cents, what is a pound worth ?
55.   If 1 sixth of a bushel of apples is worth 10 cents, what is a bushel worth ?
56.   If 1 twelfth of a vessel is worth $500, what is the whole vessel worth ?
57.   If 2 thirds of a melon are worth 8 cents, what is 1 third of it worth ?   What is the whole worth ?

ANALYSIS. — *If* 2 *thirds of a melon are worth* 8 *cents,* 1 *third of it is worth* 1 *half of* 8 *cents, which is* 4 *cents ; and if* 1 *third of a melon is worth* 4 *cents,* 3 *thirds, or the whole melon, is worth* 3 *times* 4 *cents, or* 12 *cents.*

58.   If 5 sixths of a barrel of sugar are worth $15, what is 1 sixth of a barrel worth ?   What is 1 barrel worth ?
59.   If 7 twelfths of an ounce of silver are worth 70 cents, what is an ounce worth ?
60.   When 3 tenths of a bushel of corn are worth 18 cents, what is a bushel worth ?

61.   3 is 1 half of what number ?

ANALYSIS. — *Since* 1 *half of some number is* 3, 2 *halves, or the number itself, is* 2 *times* 3, *or* 6. *Therefore* 3 *is* 1 *half of* 6.

62.   7 is 1 seventh of what number ?

63.   12 is 1 ninth of what number ?

64.   4 is 2 thirds of what number ?

ANALYSIS. — *Since* 2 *thirds of some number is* 4, 1 *third of that number is* 1 *half of* 4, *or* 2; *and* 3 *thirds, or the number itself, is* 3 *times* 2, *or* 6. *Therefore* 4 *is* 2 *thirds of* 6.

65.   8 is 4 sevenths of what number ?

66.   10 is 5 twelfths of what number ?

LESSON XXVI.

1.   If a pear is worth 2 cents, what part of the pear is worth 1 cent?

2.   1 is what part of 2 ?

ANS.   1 *is* 1 *half of* 2; *because* 1 *taken* 2 *times equals* 2.

3.   1 is what part of 4 ?   Of 3 ?   Of 8 ?   Of 7 ? Of 10 ?   Of 9 ?   Of 13 ?   Of 21 ?

4.   1 cent is what part of 6 cents ?   Of 5 cents ? Of 12 cents ?   Of 20 cents ?   Of 25 cents ?

5.   3 is what part of 6 ?

ANS.   3 *is* 1 *half of* 6; *because* 3 *is contained in* 6 2 *times.*

6.   5 apples are what part of 10 apples ?   Of 15 apples ?   Of 25 apples ?   Of 20 apples ?

7.   8 is what part of 24 ?   Of 64 ?   Of 56 ?

8.   What part of 30 is 3 ?   5 ?   6 ?   10 ?   2 ? 15 ?   What part of 60 is 30 ?

9.   2 is what part of 6 ?   4 is what part of 12 ?

10.   If a barrel of flour costs $ 10, what part of a barrel will cost $ 2 ?

ANALYSIS. — *If* $ 10 *buy* 1 *barrel of flour,* $ 2 *will buy the same part of a barrel that* $ 2 *are of* $ 10, *or* 1 *fifth.*

11.   If 12 cherries cost 1 cent, what will 4 cherries be worth ?

12.,   When a pound of spice can be bought for 24 cents, what part of a pound can be bought for 4 cents ?

13.   If $ 14 will buy a barrel of beef, what part of a barrel will $ 2 buy ? $ 7 ?

14.   If 3 bushels of cranberries cost $ 12, what part of $ 16 will 1 bushel cost ?

15.   When 5 quarts of cherries can be bought for 35 cents, what part of 21 cents will 1 quart cost ?

16.   If an orange be worth 3 cents, how much of it is worth 1 cent ?   2 cents ?

ANALYSIS. — *If* 3 *cents buy* 1 *orange,* 1 *cent will buy* 1 *third of an orange ; and if* 1 *cent buy* 1 *third of an orange,* 2 *cents will buy* 2 *times* 1 *third, or* 2 *thirds, of an orange.*

17.   If 4 dimes will purchase a yard of flannel, what part of a yard will 1 dime purchase ?   3 dimes ?

18.   At the rate of $ 6 a cord, how much wood can be bought for $ 5 ?

19.   If 11 pea-nuts are worth 1 cent, how much are 3 pea-nuts worth ?

20.   3 is what part of 4 ?

ANALYSIS.   1 *is* 1 *fourth of* 4, *and* 3 *is* 3 *times* 1 *fourth, or* 3 *fourths, of* 4.

21.   5 is what part of 6 ?   Of 8 ?   Of 12 ?   Of 9 ?

22.   What part of 5 is 3 ?   2 ?   4 ?

23.   What part of 17 is 5 ?   7 ?   4 ?   11 ?

24.   7 is what part of 8 ?   Of 10 ?   Of 16 ?   Of 20 ?

25.   3 apples are what part of 7 apples ?   Of 10 apples ?   Of 11 apples ?   Of 22 apples ?

26. 8 are how many times 3?

ANALYSIS. *8 are 2 times 3, with a remainder of 2, which is 2 thirds of 3. Therefore 8 are 2 times 3, and 2 thirds of 3.*

27. 7 are how many times 2? 3? 4?

28. How many times is 2 contained in 13? In 25? In 17? In 19? In 21?

29. How many times is 3 contained in 4? In 8? In 16? In 20? In 25? In 38?

30. How many times 4 in 5? In 10? In 15?

31. How many times 5 in 8? In 9? In 11? In 29? In 31? In 47? In 49?

32. How many times 6 in 7? In 15? In 35?

33. How many times 7 in 17? In 46? In 78?

34. At $9 a barrel, how many barrels of flour can be bought for $64?

35. At 8 cents a pound, how many pounds of rice can be bought for 78 cents?

36. In how long a time will a ship sail 93 miles, at the rate of 10 miles an hour?

37. If 11 pea-nuts cost 1 cent, how much will 100 pea-nuts cost?

38. If strawberries are 6 cents a pint, how many quarts can be bought for 62 cents?

39. How many dozen in 105? In 111?

40. 2 times 2, and 1 half of 2, are how many?

SOLUTION. *2 times 2 are 4, and 1 half of 2 is 1, which added to 4 makes 5.*

41. 2 times 3, and 1 third of 3, are how many?

42. 10 times 5, and 1 fifth of 5, are how many?

43. 6 times 7, and 1 seventh of 7, are how many?

44. 4 times 12, and 7 twelfths of 12, are how many?

SOLUTION. *4 times 12 are 48; 1 twelfth of 12 is 1, and 7 twelfths are 7 times 1, which are 7; 48 and 7 are 55.*

45.   5 times 3, and 2 thirds of 3, are how many ?

46.   5 times 9, and 5 ninths of 9, are how many ?

47.   10 times 10, and 9 tenths of 10, are how many ?

48.   What is 1 third of 2 ?

ANALYSIS.   1 *third of* 1 *is* 1 *third, and* 1 *third of* 2 *is* 2 *times* 1 *third, which are* 2 *thirds.*

NOTE. — When no other denomination is expressed or implied, 2 thirds is understood to mean 2 thirds of 1.

49.   What is 1 fifth of 3 ?   Of 4 ?   Of 2 ?

50.   What is 1 tenth of 3 ?   Of 2 ?   Of 4 ?   Of 7 ?   Of 9 ?   Of 5 ?   Of 6 ?   Of 8 ?

51.   What is 1 seventh of 5 apples ?

ANS.   5 *sevenths of an apple.*

52.   What is 1 ninth of 4 oranges ?   Of 7 oranges ? Of 8 oranges ?   Of 5 oranges ?

53.   If 5 copies of a book are worth 4 dollars, what is 1 copy worth ?

54.   If 6 yards of cloth are worth $5, what part of a dollar is 1 yard worth ?

55.   When 15 cents will buy 4 yards of ribbon, what part of a yard will 1 cent buy ?

56.   When 4 barrels of flour cost $29, what part of a barrel can be bought for $1 ?

57.   What is 1 third of 5 ?

ANALYSIS.   1 *third of* 5 *is* 1, *with a remainder of* 2, *and* 1 *third of* 2 *is* 2 *thirds.   Therefore* 1 *third of* 5 *is* 1 *and* 2 *thirds.*

58.   What is 1 half of 3 ?   Of 5 ?   Of 7 ?   Of 11 ?   Of 19 ?   Of 23 ?   Of 31 ?

59.   What is 1 third of 4 ?   Of 7 ?   Of 10 ?   Of 11 ?   Of 17 ?   Of 22 ?   Of 26 ?

60.   What is 1 fourth of 21 ?   Of 25 ?   Of 30 ? Of 35 ?   Of 42 ?   Of 47 ?

61. What is 1 eighth of 21 apples?

ANS. *2 apples and 5 eighths of an apple.*

62. What is 1 tenth of 13 oranges? Of 24 oranges? Of 73 oranges?

63. 5 is 2 times what number?

ANALYSIS. — *If* 2 *times some number is* 5, *once that number, or the number itself, is* 1 *half of* 5, *which is* 2 *and* 1 *half.*

64. 11 is 2 times what number? 3 times? 4 times? 5 times?

65. 23 is 8 times what number? 5 times? 4 times? 7 times? 6 times?

66. 25 nuts are 12 times what number of nuts?

67. How can you divide 3 apples equally between 2 persons?

68. If 9 pounds of meat are worth 95 cents, what is 1 pound worth?

69. At $7 a yard, what costs 1 sixth of a yard of broadcloth?

70. If 1 man can do a piece of work in 20 days, in how many days can 3 men do it?

LESSON XXVII.

1. 8 is 1 fourth of what number?
2. 8 is 1 third of what number?
3. 7 is 1 fifth of what number?
4. 9 is 1 fourth of what number?
5. 12 is 1 third of what number?
6. 15 is 1 sixth of what number?
7. 18 is 1 tenth of what number?
8. 6 is 3 fourths of what number?
9. 12 is 2 fifths of what number?
10. 8 is 4 fifths of what number?

11.   9 is 3 sixths of what number?
12.   10 is 5 eighths of what number?
13.   16 is 4 ninths of what number?
14.   18 is 9 tenths of what number?
15.   20 is 4 fifths of what number?
16.   24 is 6 eighths of what number?
17.   30 is 3 elevenths of what number?
18.   14 is 7 eighths of how many times 4?
19.   9 is 3 fourths of how many times 6?
20.   10 is 5 sixths of how many times 4?
21.   24 is 3 ninths of how many times 12?
22.   20 is 4 fifths of how many times 5?
23.   24 is 4 fifths of how many times 10?
24.   18 is 3 ninths of how many times 6?
25.   30 is 3 fifths of how many times 10?
26.   28 is 4 sevenths of how many times 7?
27.   24 is 4 twelfths of how many times 9?
28.   15 is 1 fourth of 5 times what number?
29.   18 is 1 third of 6 times what number?
30.   12 is 1 sixth of 8 times what number?
31.   22 is 1 half of 11 times what number?
32.   30 is 1 third of 15 times what number?
33.   16 is 2 sevenths of 8 times what number?
34.   9 is 3 ninths of 9 times what number?
35.   36 is 6 tenths of 20 times what number?
36.   5 eighths of 32 are how many times 5?
37.   6 eighths of 48 are how many times 12?
38.   7 ninths of 54 are how many times 6?
39.   4 tenths of 70 are how many times 7?
40.   3 elevenths of 44 are how many times 6?
41.   5 twelfths of 84 are how many times 7?
42.   8 sevenths of 63 are how many times 8?
43.   4 times 3, and 2 thirds of 3, are how many?
44.   5 times 4, and 3 fourths of 4, are how many?
45.   7 times 6, and 4 sixths of 6, are how many?
46.   8 times 4, and 3 fourths of 4, are how many?

47. 5 times 7, and 4 sevenths of 7, are how many?
48. 9 times 8, and 7 eighths of 8, are how many?
49. 6 times 9, and 5 ninths of 9, are how many?
50. 5 times 9, and 3 ninths of 9, are how many?
51. 8 times 10, and 9 tenths of 10, are how many times 9? How many times 11?
52. 7 times 7, and 5 sevenths of 7, are how many times 6? How many times 8?
53. 9 times 12, less 11 twelfths of 12, are how many times 10? How many times 9?
54. 10 times 11, less 7 elevenths of 11, are how many times 8? How many times 12?
55. 2 thirds of 9 is 1 fourth of what number?
56. 3 fourths of 12 is 1 third of what number?
57. 4 fifths of 15 is 3 fifths of what number?
58. 5 ninths of 36 is 2 fifths of how many times 10? How many times 25?
59. 4 tenths of 30 is 2 thirds of how many times 3? How many times 9?
60. 5 sixths of 18 is 3 fifths of how many times 5? How many times 4?
61. 6 tenths of 20 is 1 seventh of what number?
62. 5 twelfths of 60 are how many times 1 sixth of 30? How many times 1 half of 10?
63. If 5 is 1 sixth of some number, what is 1 tenth of the same number?
64. If 8 is 1 fifth of some number, what is 3 fourths of the same number?
65. If 18 is 6 sevenths of some number, what is 2 thirds of the same number?
66. If 20 is 4 ninths of some number, what is 3 fifths of the same number?
67. If 24 is 8 tenths of some number, what is 5 sixths of the same number?
68. 12 times 8, and 4 eighths of 8, are 11 times what number?

4

LESSON XXVIII.

1.˙ If a melon costs 8 cents, what will 1 fourth of a melon cost?

2. At $5 a yard, what will 1 fifth of a yard cost?

3. At $6 a ton, what will 7 sixths of a ton cost?

4. At $12 a barrel, what will 1 barrel and 1 sixth of a barrel of flour cost?

5. When a man is working for $15 a week, how much does he earn in 2 weeks and 1 fifth of a week?

ANALYSIS. — *If in* 1 *week a man earn* $15, *in* 2 *weeks he will earn* 2 *times* $15, *which are* $30, *and in* 1 *fifth of a week he will earn* 1 *fifth of* $15, *which is* $3; $3 *added to* $30 *make* $33. *Therefore, in* 2 *weeks and* 1 *fifth of a week, at the rate of* $15 *a week, a man will earn* $33.

6. At 16 cents a pound, what cost 3 pounds and 3 fourths of a pound of sugar?

7. At $8 a cord, what cost 4 cords and 3 fourths of a cord of wood?

8. At 6 dimes a bushel, what cost 10 bushels and 2 thirds of a bushel of corn?

9. A teacher, being asked how many scholars he had, replied that the smallest number that ever had been present was 18, which was just 3 ninths of his whole number; how many had he?

10. Sold a horse for $150, which was 5 thirds of what he cost; what did he cost?

11. I have $4, which is 1 fifth of what I divided among 10 men; how much did each receive? ·

12. William has 63 cents in his pocket, which is 9 fifths of what he has in his money-box; how much has he in his money-box?

13. A boy, being asked how many chickens he had, answered that his largest brood contained 20, which was 4 fifths of the whole number, and the

whole number was 5 times as many as he had in the smallest brood; how many had he, and how many were there in the smallest brood?

14. A farmer had 42 sheep, and sold 5 sevenths of them to 5 of his neighbors, each receiving an equal number; how many did each receive?

15. I had 24 cherries, but have divided 5 sixths of them equally among 10 children; how many does each receive?

16. A gentleman had 36 pears, and gave 1 sixth of them to Frank, and divided the remainder equally among his 5 brothers; how many did Frank receive more than each of his brothers?

17. At 10 cents a pound, what cost 20 pounds and 3 tenths of a pound of raisins?

18. At 18 cents a pound, what cost 8 pounds and 5 ninths of a pound of butter?

19. Henry gave $5 for a vest, which was just 1 half of 4 times as much as he gave for a hat; how much did he give for the hat?

20. What cost 5 barrels of flour, if 1 tenth of a barrel cost 9 dimes?

21. John has 20 cents, which is 4 fifths of 5 times as many as Joseph has; how many has Joseph?

22. Emma is 12 years old, and 7 sixths of her age is just 7 eighths of her sister's age; what is her sister's age?

23. A pier of a certain bridge stands 10 feet in the water, which is 2 fifths of the height of the pier lacking 5 feet; what is the height of the pier?

24. Bought 4 pounds and 3 fourths of a pound of rice at 8 cents a pound, and paid for it with berries at 5 cents a quart; how many quarts did it take?

25. Bought a watch for $40, and a chain for $5 more than 2 fifths the cost of the watch; what did both cost?

26. Sold a hogshead of molasses for $ 36, which was 9 eighths of what it cost; what did it cost?

27. A farmer has 6 cows, and 2 thirds of the number of cows is equal to 1 fourth of the number of his sheep; how many sheep has he?

28. Four boys wished their father to divide 50 cents equally among them. " No," said he; " but I will give you 4 fifths of it, and 1 half of the remaining fifth to the one that will tell how to divide it"; how much would the one who could tell receive?

29. If 6 pounds of sugar cost 96 cents, what cost 5 eighths of a pound?

30. A man, wishing to draw the water from a well containing 120 gallons, found that while he could draw out 12 gallons in a minute, 2 thirds as many gallons would run in during the same time; how long would it take him to exhaust the well?

31. Bought 9 yards of broadcloth for $ 36, which was 4 fifths of its real value; what was its real value per yard?

32. James, in reciting, had given in one day 8 imperfect answers, and 3 fourths of the imperfect answers were just 2 thirds of the number of perfect answers; how many perfect answers had he given?

33. In a small basket there are 12 pears, and 2 thirds of these are 2 sixths of the number in a large basket; if the whole be divided equally among 4 boys, how many will each receive?

34. If 10 copies of a book cost $ 12, what will 5 copies cost? (See Appendix, Les. XVIII. Ex. 25.)

35. When 8 cents will buy 12 apples, how many apples will 2 cents buy?

36. If 9 hats cost $ 24, how many hats can be bought for $ 8?

37. If 13 pounds of veal cost 91 cents, what part of 42 cents will 2 pounds cost?

38. Bought a horse for 120 dollars, and sold him at 1 sixth more than he cost; for how much was he sold?

39. James has 48 apples, and his brother has 3 fourths as many; how many has his brother?

40. Bought a horse for $160, and a wagon for $5 more than 3 fourths the cost of the horse; what was the cost of the wagon?

41. When 35 cents will buy a pound of butter, how many cents will buy 3 sevenths of a pound? 5 sevenths of a pound?

42. A boy at recitation having answered 4 questions wrong, found the number to be just 2 sevenths of all the questions asked; how many were asked?

43. If a piece of land is worth $75, how many times $5 is 1 fifth of it worth?

44. Edward has 48 walnuts, and Alfred has 5 eighths as many; how many has Alfred?

45. What cost 3 tenths of a ton of hay, at 7 dimes a hundred-weight?

46. William earns 63 cents a day, and John 10 ninths as many and 5 cents more; how many cents does John earn?

47. What cost 2 ninths of a hogshead of molasses, at 10 cents a quart?

48. What cost 5 twelfths of a gross of writing-books, at 10 cents a book?

49. If 2 thirds of a pound of sugar cost 8 cents, what will 3 fourths of a pound cost?

50. Robert, having $60 to spend, paid 1 twelfth of it for a hat, 2 fifths for a suit of clothes, and 3 tenths for tuition; how much had he left?

51. Frank had 2 bags filled with nuts, and his brother wished to buy 1 of the bags. "William," said Frank, "I have 9 quarts in the smaller bag, and 2 thirds of that number is just 1 half the number of

quarts in the larger bag; and if you will tell me the number of quarts in the larger bag, I will give them to you; how many were there in both bags?

52. A certain school is divided into three classes; in the first class there are 54 pupils, which are 9 tenths of the number in the second class, and the number in the second class is equal to 6 fifths of 2 times the number in the third class; how many are there in each of the classes?

LESSON XXIX.

Such expressions as one half, one third, two thirds, &c., may be represented by figures, thus:

$\frac{1}{2}$,	one half.	$\frac{3}{5}$,	three fifths.
$\frac{1}{3}$,	one third.	$\frac{4}{5}$,	four fifths.
$\frac{2}{3}$,	two thirds.	$\frac{1}{6}$,	one sixth.
$\frac{1}{4}$,	one fourth.	$\frac{5}{6}$,	five sixths.
$\frac{2}{4}$,	two fourths.	$\frac{3}{7}$,	three sevenths.
$\frac{3}{4}$,	three fourths.	$\frac{9}{8}$,	nine eighths.
$\frac{1}{5}$,	one fifth.	$\frac{11}{12}$,	eleven twelfths.
$\frac{2}{5}$,	two fifths.	$\frac{13}{19}$,	thirteen nineteenths.

The figure or figures above the short horizontal line are called the *numerator* of the fraction, and the figure or figures below the line are called the *denominator*.

The denominator shows into how many equal parts a unit has been divided, and the numerator shows how many of these equal parts have been taken. The numerator and denominator are called the *terms* of the fraction.

When the numerator is less than the denominator, the fraction is called a *proper fraction;* when the numerator is equal to or larger than the denominator, the fraction is called an *improper fraction.* A fraction with a whole number is called a *mixed number.*

1. What kind of a fraction is $\frac{1}{2}$? What is the 2 called? What does it show?

2. Into how many equal parts is the unit divided, to give the fraction $\frac{1}{3}$? How many of these equal parts are taken?

3. What kind of a fraction is $\frac{5}{7}$? Why?

4. In the fraction $\frac{2}{3}$, what is the 2 called? What does it show?

5. In the fraction $\frac{7}{9}$, what is the 9 called? Into how many equal parts does it show the thing to be divided?

6. What kind of a fraction is $\frac{4}{3}$? Why?

7. In the fraction $\frac{8}{7}$, what does the 7 show? What does the 8 show?

8. What is the expression $5\frac{2}{11}$ called? Why?

Note. $5\frac{2}{11}$ is read five and two elevenths.

9. Into how many equal parts is the unit divided to give the fraction $\frac{5}{4}$? How many of the parts are taken?

10. How can you find $\frac{3}{5}$ of anything?

11. How can you find $\frac{5}{9}$ of anything?

12. In 12 how many fourths?

Note. — For forms of Analysis, see Lesson **XXV.**

13. Reduce 15 to thirds. To fifths.

14. How can you reduce a whole number **to a** fraction having any given denominator?

15. How many times $\frac{1}{4}$ in 5? In 16?

16. How many times $\frac{1}{6}$ in 7? In 10?

17. How many times $\frac{1}{7}$ in 9? In 12?

18. In $6\frac{2}{3}$ how many thirds?

19. Reduce $7\frac{2}{5}$ to an improper fraction.

Note. — That is, change 7 and 2 fifths to fifths.

20. Reduce $9\frac{5}{6}$ to an improper fraction.

21. Express $12\frac{1}{4}$ by an improper fraction.
22. Express $18\frac{5}{10}$ by an improper fraction.
23. Express $11\frac{5}{11}$ by an improper fraction.
24. How do you change a mixed number to an improper fraction?
25. How many times 1 in $\frac{8}{4}$? In $\frac{11}{4}$?
26. How many times 1 in $\frac{32}{16}$? In $\frac{54}{18}$?
27. How many times 1 in $\frac{42}{14}$? In $\frac{56}{14}$?
28. How many times 1 in $\frac{63}{9}$? In $\frac{72}{9}$?
29. How many times 1 in $\frac{108}{12}$? In $\frac{96}{12}$?
30. How many times 1 in $\frac{126}{10}$? In $\frac{84}{10}$?
31. How many times 1 in $\frac{96}{4}$? In $\frac{88}{4}$?
32. How many times 1 in $\frac{92}{2}$? In $\frac{102}{2}$?
33. How many times 1 in $\frac{85}{3}$? In $\frac{66}{3}$?
34. How many times 1 in $\frac{42}{7}$? In $\frac{84}{7}$?
35. How many times 1 in $\frac{101}{4}$? In $\frac{84}{4}$?
36. Reduce $\frac{32}{8}$ to an equivalent whole number.
37. Reduce $\frac{8}{5}$ to an equivalent mixed number.
38. Reduce $\frac{12}{7}$ to an equivalent mixed number.
39. Reduce $\frac{17}{3}$ to an equivalent mixed number.
40. Reduce $\frac{25}{8}$ to an equivalent mixed number.
41. Reduce $\frac{99}{11}$ to an equivalent whole number.
42. Reduce $\frac{104}{3}$ to an equivalent whole number.
43. Reduce $\frac{175}{25}$ to an equivalent whole number.
44. Express $\frac{80}{15}$ by an equivalent whole number.
45. Express $\frac{100}{24}$ by an equivalent mixed number.
46. Express $\frac{168}{40}$ by an equivalent mixed number.
47. Express $\frac{146}{12}$ by an equivalent mixed number.
48. Express $\frac{131}{10}$ by an equivalent mixed number.
49. Express $\frac{126}{60}$ by an equivalent mixed number.
50. How do you reduce an improper fraction to an equivalent whole or mixed number?

51. Reduce $\frac{4}{6}$ to thirds.

ANALYSIS. — *Since 1 is equal to $\frac{6}{6}$, $\frac{1}{3}$ of 1 is equal to $\frac{1}{3}$ of $\frac{6}{6}$, which is $\frac{2}{6}$; and if $\frac{2}{6}$ are equal to $\frac{1}{3}$, $\frac{4}{6}$ are equal to as many thirds as 2 sixths are contained times in 4 sixths, which are 2. Therefore $\frac{4}{6} = \frac{2}{3}$.*

NOTE. — It will be seen that the same result can be reached more readily by dividing both terms of the fraction by 2. *Dividing the numerator and denominator of a fraction by the same number does not alter the value of the fraction.*

52. Reduce $\frac{6}{12}$ to sixths. To fourths.

NOTE. — A fraction is said to be reduced to lower terms when it is changed to an equivalent fraction having smaller terms. A fraction is in *its lowest terms* when no number greater than 1 will divide both its numerator and denominator, without a remainder.

53. Reduce $\frac{6}{12}$ to its lowest terms.

54. How many fifths in $\frac{9}{15}$? In $\frac{8}{10}$?

55. Reduce $\frac{4}{12}$ to sixths. To its lowest terms.

56. Reduce $\frac{12}{18}$ and $\frac{16}{24}$ to sixths. To their lowest terms.

57. How many thirds in $\frac{10}{15}$? In $\frac{10}{30}$?

58. Reduce $\frac{5}{10}$ and $\frac{7}{14}$ to their lowest terms.

59. Reduce $\frac{9}{27}$ and $\frac{10}{18}$ to ninths.

60. Reduce $\frac{5}{15}$ and $\frac{8}{32}$ to their lowest terms.

61. Reduce $\frac{5}{25}$ and $\frac{24}{48}$ to their lowest terms.

62. Reduce $\frac{25}{60}$ and $\frac{27}{72}$ to their lowest terms.

63. Reduce $\frac{24}{96}$ and $\frac{64}{84}$ to their lowest terms.

64. Reduce $\frac{60}{80}$ and $\frac{25}{100}$ to their lowest terms.

65. Reduce $\frac{60}{70}$ and $\frac{75}{200}$ to their lowest terms.

66. Reduce $\frac{24}{84}$ and $\frac{56}{63}$ to their lowest terms.

67. When is a fraction in its lowest terms ?

68. How do you reduce a fraction to its lowest terms ?

69. Why is the value of a fraction not changed by reducing it to its lowest terms ?

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70.   Reduce ⅔ to sixths.

ANALYSIS. — *Since* 1 *is equal to* 3/3, *½ of* 1 *is equal to ½ of*
3/3, *which is* 3/6; *and if ½ is equal to* 3/6, *⅔ are equal to* 2 *times*
3/6, *which are* 6/6.   *Therefore* ⅔ = 6/6.

NOTE. — It will be seen that the same result may also be
reached by multiplying both terms of the fraction by 2.   *Multi-
plying the numerator and denominator of a fraction by the same
number does not alter the value of the fraction.*

When a fraction is changed to an equivalent one having larger
terms, it may be said to be reduced to *higher terms.*

71.   Reduce ½ to tenths.

72.   Reduce ¼ and ⅓ to twelfths.

73.   How many eighths in ¾?   In ½?

74.   How many sixths in ⅔?   In ½?

75.   How many twenty-fifths in ⅘?   In ⅗?

76.   In ⅚ how many fifty-fourths?   ·

77.   In 7/12 how many ninety-sixths?

78.   Reduce ½ and 2/11 to forty-fourths.

79.   How do you reduce a fraction to higher
terms?

80.   Why is the value of a fraction not changed
by multiplying both terms by the same number?

LESSON XXX.

1.   Gave ¾ of an apple to one boy, ¾ to another,
and ¾ to another; how many fourths were given
away?   How many whole apples?

NOTE. — Fractions are said to have a *common denominator*
when their denominators are alike.

2.   Lydia has ⅗ of a dollar, and Mary ⅖ of a dol-
lar; how many dollars have they both?

3.   Sold ⅚ of an acre of land to one man, ⅚ to
another, and ⅔ to another; how many acres were
sold?

4. $\frac{1}{6} + \frac{2}{6} + \frac{5}{6} + \frac{4}{6}$ are how many times 1 ?

5. Edmund had $7\frac{3}{4}$ dollars, and his father gave him $\frac{3}{4}$ of a dollar more; how many dollars had he then ?

NOTE. $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$; and $7 + 1\frac{1}{2} = 8\frac{1}{2}$.

6. Bought a barrel of flour for $ $9\frac{5}{8}$, and a yard of velvet for $ $4\frac{7}{8}$; how much did the whole cost ?

7. James gathered $8\frac{1}{8}$ quarts of berries, Frank $4\frac{5}{8}$ quarts, and Arthur $6\frac{1}{8}$ quarts; how many did they all gather ?

8. $5\frac{3}{10} + 7\frac{9}{10} + 12\frac{8}{10}$ are how many times 1 ?

9. $3\frac{1}{5} + 4\frac{3}{5} + \frac{6}{5}$ are how many times 1 ?

10. $8\frac{3}{8} + 4\frac{7}{8} + 4$ are how many times 1 ?

11. A man who owned $\frac{7}{8}$ of a ship sold $\frac{5}{8}$ of it; what part of the ship did he then own ?

12. If $\frac{7}{12}$ of an acre are sold from a piece of land containing $2\frac{11}{12}$ acres, how much land remains ?

13. George had $ $17\frac{3}{8}$, but has spent $ $2\frac{3}{8}$ of it; how much has he left ?

14. From $25\frac{7}{12}$ acres of land there have been sold $4\frac{5}{12}$ acres; how much is left ?

NOTE. — Answers should usually be expressed in their simplest form; hence fractions should be reduced to their lowest terms, and improper fractions to mixed numbers.

15. If you should give $\frac{4}{8}$ of a melon to your brother, and retain the rest for yourself, what part would you retain ?

NOTE. $1 = \frac{8}{8}$; and $\frac{8}{8} - \frac{4}{8} = \frac{4}{8}$.

16. A gentleman owned a ship, but has sold $\frac{5}{16}$ of it; what part does he still own ?

17. Sarah bought 3 yards of cloth, and gave Ellen $\frac{3}{4}$ of a yard; how much had she left ?

18. Bought a Bible for $ 5, less $ $\frac{7}{8}$; how much was paid for it ?

19. From a hogshead of wine there leaked out $6\frac{3}{8}$ gallons; how many gallons remained ?

20. $75 - 12\frac{4}{5}$ are how many times 1 ?

21. $13\frac{1}{7} - 10\frac{6}{7}$ are how many times 1 ?

NOTE. $13\frac{1}{7} = 12\frac{8}{7}$; and $12\frac{8}{7} - 10\frac{6}{7} = 2\frac{2}{7}$.

22. $82\frac{1}{6} - 4\frac{5}{6}$ are how many times 1 ?

23.   George put into the bank at one time $ 11, and at another time $ $6\frac{5}{12}$; how much more must he put in to make up $ 20 ?

24.   Three men bought a boat together; A paid $ $18\frac{4}{10}$, C $ $20\frac{7}{10}$, and B the balance ; how much did B pay, if the boat cost $ 50 ?

25.   A farmer had 60 bushels of apples, but sold at one time $16\frac{3}{8}$ bushels, and at another time $30\frac{7}{8}$ bushels ; how many had he left ?

26. $10 + 6\frac{1}{3} - 9\frac{2}{3}$ are how many ?

27. $20\frac{4}{5} + 10\frac{3}{5} - 6\frac{2}{5}$ are how many ?

28. $21\frac{4}{11} + 7\frac{8}{11} - 10\frac{1}{11}$ are how many ?

29. $16\frac{1}{3} + 8\frac{2}{3} - 20\frac{1}{3}$ are how many ?

30.   Bought a coat for $ $11\frac{5}{8}$, and a vest for $ $3\frac{7}{8}$, and gave in payment two ten-dollar bills ; how much change should be received back ?

31.   George is $11\frac{3}{12}$ years old, Simeon $8\frac{7}{12}$, and Edward $6\frac{3}{12}$; how much does the sum of their ages exceed 5 times 5 years ?

32.   Sold 6 cords of wood, at $ 5 a cord, and bought a hundred-weight of sugar for $ $14\frac{3}{4}$, and a thousand feet of boards for $ $15\frac{3}{4}$; how much more did the articles bought cost than was received for the wood ?

33.   Sold a cow for $ 40, and took in part payment a plow worth $ $11\frac{2}{3}$, and a harness worth $ $20\frac{2}{3}$; how much was then due ?

34.   Bought 3 barrels of beef, at $ 14 a barrel, and paid in merchandise $ $24\frac{7}{16}$, and the balance in cash ; how much less was paid in cash than in merchandise ?

LESSON XXXI.

1. A man bought $\frac{1}{3}$ of an acre of land at one time, and $\frac{2}{3}$ of an acre at another time; how many thirds of an acre did he buy?

2. Gave $\frac{1}{2}$ of a bushel of corn to one man, and $\frac{1}{3}$ to another; how many sixths did I give them both?

NOTE. — The methods of reducing fractions to higher and lower terms will be found in Lesson XXIX.

3. Gave $\frac{1}{2}$ of a dollar to Lydia, and $\frac{3}{4}$ to Sarah; to which was the most given?

4. A man divided a quantity of fruit among 3 of his friends; he gave $\frac{1}{2}$ to one, $\frac{1}{4}$ to another, and $\frac{3}{8}$ to another; how many fourths did each receive?

5. $\frac{1}{2} + \frac{1}{4} + \frac{3}{8}$ are how many fourths?

6. A gentleman kept 3 fires in his house, during the winter; the first fire consumed $\frac{3}{8}$ of a ton of coal, the second $\frac{1}{2}$, and the third $\frac{1}{6}$ of a ton; how many sixths did each consume? How many tons did they all consume?

7. $\frac{2}{3} + \frac{1}{2} + \frac{1}{6}$ are how many sixths? How many times 1?

8. Reduce $\frac{1}{2}$ and $\frac{3}{4}$ to a common denominator.

NOTE. — When the given fractions are expressed in their lowest terms, such a denominator must be chosen as will contain each of the given denominators. A common denominator can always be obtained by multiplying the given denominators together, but it will not always be the *least* one that can be used.

9. Reduce $\frac{1}{4}$ and $\frac{2}{5}$ to a common denominator.

10. Reduce $\frac{1}{4}$, $\frac{4}{5}$, and $\frac{5}{8}$ to a common denominator.

11. Change $\frac{3}{4}$ and $\frac{3}{12}$ to a common denominator.

12. Change $\frac{2}{8}$ and $\frac{3}{12}$ to fourths.

13. Change $\frac{3}{4}$ and $\frac{5}{6}$ to a common denominator.

14. A farmer sold $\frac{3}{4}$ of a bushel of peaches to

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one man, $\frac{2}{5}$ to another, and $\frac{2}{10}$ to another; how many bushels did he sell in all?

NOTE. — Fractions having different denominators cannot be added together till they are reduced to a common denominator.

15. What is the sum of $\frac{1}{2}$ and $\frac{4}{5}$?

16. What is the sum of $\frac{3}{4}$ and $\frac{5}{6}$?

17. What is the sum of $\frac{4}{5}$ and $\frac{7}{8}$?

18. What is the sum of $\frac{7}{9}$ and $\frac{7}{6}$?

19. $\frac{5}{4} + \frac{4}{5}$ are how many times 1?

20. $\frac{3}{11} + \frac{4}{3}$ are how many times 1?

21. $1\frac{6}{7} + 2\frac{1}{2}$ are how many times 1?

22. $11\frac{1}{4} + 10\frac{1}{8}$ are how many times 1?

23. What is the sum of $\frac{1}{2} + \frac{1}{6} + \frac{9}{5}$?

24. What is the sum of $\frac{4}{5} + \frac{3}{10} + \frac{3}{4}$?

25. What is the sum of $3\frac{4}{9} + \frac{1}{3} + \frac{4}{6}$?

26. What is the sum of $11 + \frac{1}{2} + \frac{7}{8}$?

27. $\frac{3}{4}$ from $\frac{7}{8}$ leave how many?

28. $\frac{1}{5}$ from $\frac{3}{4}$ leave how many?

29. $\frac{2}{3}$ from $1\frac{2}{5}$ leave how many?

30. $\frac{5}{6}$ from $1\frac{7}{2}$ leave how many?

31. $10\frac{1}{5}$ less $1\frac{2}{3}$ are how many?

32. $1\frac{9}{2}$ less $\frac{8}{3}$ are how many?

33. $11\frac{3}{4} - 10\frac{1}{2}$ leave how many?

34. $16 - 1\frac{7}{8}$ leave how many?

35. $\frac{1}{3} + \frac{1}{4}$ are how much less than a whole one?

36. $\frac{1}{8} + \frac{1}{2}$ are how much less than a whole one?

37. $\frac{1}{5} + \frac{2}{8} + \frac{3}{4}$ are how much less than 2?

38. When have fractions a common denominator?

39. How do you reduce fractions to a common denominator?

40. Does multiplying or dividing both the numerator and denominator by the same number alter the value of a fraction?

LESSON XXXII.

1. If a family consume $\frac{2}{5}$ of a barrel of flour in 1 week, how much will it consume in 6 weeks?

ANALYSIS. — *If in* 1 *week a family consume* $\frac{2}{5}$ *of a barrel of flour, in* 6 *weeks it will consume* 6 *times* $\frac{2}{5}$ *of a barrel, which are* $\frac{12}{5}$ *of a barrel, or* $2\frac{2}{5}$ *barrels.*

2. What cost 7 yards of cloth, at $\frac{4}{5}$ of a dollar a yard?

3. At $\frac{2}{3}$ of a cent apiece, what cost 12 eggs?

4. At $\frac{3}{4}$ of a dollar a day, how much can be earned in 12 days?

5. If a man can walk $3\frac{1}{2}$ miles in an hour, how far can he walk in 10 hours?

NOTE. — Either multiply the whole number and the fraction separately, or reduce to an improper fraction before multiplying.

6. What cost 7 chairs, at $5\frac{1}{2}$ dollars apiece?

7. At $\frac{1}{5}$ of a dollar a peck, what cost 5 pecks of apples? 7 pecks? 8 pecks?

8. At $\frac{7}{16}$ of a dollar a bushel, what cost 3 bushels of potatoes? 8 bushels? 12 bushels?

9. How many are 4 times $\frac{1}{3}$? 5 times $\frac{4}{5}$?

10. How many are 6 times $\frac{3}{5}$? 7 times $\frac{2}{7}$?

11. How many are 9 times $\frac{1}{2}$? 10 times $\frac{5}{9}$?

12. At $6\frac{1}{4}$ cents a pound, what cost 9 pounds of rice? 10 pounds? 12 pounds?

13. When eggs are $16\frac{2}{3}$ cents a dozen, what cost 3 dozen? 6 dozen?

14. If a peck of corn costs $\frac{3}{16}$ of a dollar, how much will 2 bushels cost?

15. What cost 14 pounds of cheese, at $10\frac{1}{2}$ cents a pound? At $10\frac{1}{3}$ cents?

16. What cost 20 bushels of wheat, at $\$2\frac{1}{5}$ a bushel? At $\$2\frac{1}{4}$?

17. What cost 13 yards of silk, at $\$2\frac{3}{4}$ a yard?

18. At $6\frac{1}{4}$ cents a nail, what cost 4 yards of cloth?   8 yards?   10 yards?

19. How many are 4 times $2\frac{1}{2}$?   6 times $3\frac{2}{5}$?

20. How many are 6 times $5\frac{2}{7}$?   9 times $5\frac{2}{3}$?

21. How many are 8 times $4\frac{1}{2}$?   10 times $5\frac{2}{9}$?

22. How many are 7 times $8\frac{7}{8}$?   12 times $11\frac{4}{12}$?

23. If a horse can trot $9\frac{3}{4}$ miles in 1 hour, how far, at that rate, can he trot in 9 hours?

24. How much can be earned in a year, at $\$11\frac{2}{3}$ a month?   At $\$10\frac{7}{8}$ a month?

25. What cost 5 bushels of corn, at $\frac{7}{10}$ of a dollar a bushel?

NOTE.—It will be seen that we here multiply the numerator by 5, and then divide both numerator and denominator by 5, in order to reduce the fraction to its lowest terms.   The same result could more easily be obtained by merely dividing the denominator by 5; thus, 5 times $\$\frac{7}{10} = \$\frac{7}{2} = \$3\frac{1}{2}$.   *Dividing the denominator of a fraction by any number produces the same effect upon the value of the fraction as multiplying its numerator by the same number.*

26. If a man can reap $\frac{7}{8}$ of an acre in a day, how many acres can 4 men reap in the same time?

27. What cost 6 pounds of opium, at $\$4\frac{5}{8}$ a pound?   At $\$4\frac{7}{12}$ a pound?

28. How many are 8 times $5\frac{3}{16}$?   8 times $12\frac{4}{8}$?

29. How many are 9 times $10\frac{5}{9}$?   9 times $2\frac{7}{18}$?

30. How many are 10 times $10\frac{3}{20}$?   10 times $15\frac{9}{10}$?   10 times $12\frac{7}{30}$?

31. How many are 9 times $\frac{84}{63}$?   9 times $\frac{72}{54}$?

32. When $\frac{1}{6}$ of a bushel of oats is worth $\frac{1}{8}$ of a dollar, what is a bushel worth?

ANALYSIS.—*If $\frac{1}{6}$ of a bushel of oats is worth $\$\frac{1}{8}$, $\frac{6}{6}$, or 1 bushel, is worth 6 times $\$\frac{1}{8}$, which are $\$\frac{6}{8}$, or $\$\frac{3}{4}$.*

33. When $\frac{1}{4}$ of a barrel of flour is worth $\$1\frac{5}{8}$, what is a barrel worth?

34. If a man can reap $\frac{7}{16}$ of an acre in $\frac{1}{2}$ of a day, how much can he reap in a whole day?

35. If 8 men can do a piece of work in $3\frac{1}{4}$ days, how long will it take 1 man to do the same work ?

36. If $\frac{1}{2}$ of a melon is worth as much as $6\frac{2}{3}$ apples, how many apples are worth as much as 1 melon ?

37. $5\frac{1}{6}$ is $\frac{1}{5}$ of what number ? $\frac{1}{10}$ of what number ? $\frac{1}{12}$ of what number ?

38. How many times 11 are 6 times $4\frac{1}{2}$?

39. 5 times $2\frac{1}{10}$ is $\frac{1}{2}$ of what number ?

40. 2 times $3\frac{5}{8}$ is $\frac{1}{3}$ of what number ?

41. How many times 25 are 21 times $\frac{800}{63}$?

42. How do you multiply a fraction by a whole number ?

43. How many cords of wood, at $5 a cord, will pay for 8 barrels of flour, at 10\frac{5}{8}$ a barrel ?

44. How many loads of hay, at $11 a load, will pay for 5 yards of cloth, at 4\frac{9}{10}$ a yard, and a debt of 8\frac{5}{10}$?

45. If 20 pipes of a certain size can fill a cistern in $\frac{9}{10}$ of an hour, in how long will a single pipe, $\frac{1}{8}$ as large as each of them, fill the same cistern ?

LESSON XXXIII.

1. If a bushel of beans is worth $4, what is $\frac{1}{8}$ of a bushel worth ?

ANALYSIS. — *If* 1 *bushel of beans is worth* $4, $\frac{1}{8}$ *of a bushel is worth* $\frac{1}{8}$ *of* $4, *which is* $\frac{4}{8}$, *or* 1\frac{1}{2}$.

NOTE. — See Exs. 48, 57, Les. XXVI.

2. If a pound of dried apples costs 8 cents, what will $\frac{1}{4}$ of a pound cost? (Ex. 39, Les. XXV.)

3. If a barrel of beef is worth $14, what is $\frac{1}{8}$ of a barrel worth ?

4.   If a man can do a certain piece of work in 25 days, in how many days can he do ½ of it?

5.   If a ton of hay is worth $18, how much is a hundred-weight worth?

6.   If you can buy 8 pounds of sugar for $1, how many pounds can you buy for $⅙?

7.   If 4 oranges are divided equally among 5 boys, what part of an orange will each boy receive? How much will 2 boys receive?

8.   If 8 bushels of apples cost $5, what part of a dollar does 1 bushel cost? 3 bushels?

9.   If 12 chairs cost $9, what costs 1 chair? What cost 5 chairs? What cost 8 chairs?

10.   If 8 bushels of corn cost $6, what cost 3 bushels? What cost 5 bushels?

11.   What cost ⅖ of a pound of butter, at 17 cents a pound?

ANALYSIS. — *If 1 pound of butter cost 17 cents, ⅕ of a pound will cost ⅕ of 17 cents, or ¹⁷⁄₅ of a cent, and ⅖ of a pound will cost 2 times ¹⁷⁄₅ of a cent, which are ³⁴⁄₅ of a cent, or 6⅘ cents.*

12.   If 1 pound of cheese costs 12 cents, what will ¾ of a pound cost?   (Ex. 44, Les. XXV.)

13.   At 19 cents a yard, what cost ¾ of a yard of calico? What cost ⅝ of a yard?

14.   If 1 pound of sugar costs 12 cents, how much will 3¾ pounds cost?   (Ex. 5, Les. XXVIII.)

15.   If a man walks at the rate of 3 miles an hour, how far will he walk in 2⅖ hours?

16.   A hogshead of molasses was sold for $33, which was ⅞ of the cost; what was the cost?

ANALYSIS. — *As ⅞ of the cost was $33, ⅐ of the cost was ⅐ of $33, or $4⅞, and ⅞, or the cost itself, was 6 times $4⅞; 6 times $4 are $24, and 6 times $⅞ are ³⁰⁄₇, or $4²⁄₇, which added to $24 gives $28²⁄₇ as the cost.*

17.   If a man travels 20 miles in ⅚ of a day, how far will he travel in a whole day?   (Ex. 57, Les. XXV.)

18. Paid $101 for a horse, which was $\frac{4}{5}$ of his real value; what was his real value?

19. A gentleman 40 years old is $2\frac{1}{2}$ times as old as his son; how old is the son?

NOTE. "$2\frac{1}{2}$ times as old" is the same as "$\frac{5}{2}$ as old."

20. When $1\frac{1}{4}$ tons of coal are sold for $9, what is that a ton?

21. What is $\frac{1}{8}$ of 4? Of 5? Of 7? Of 12? Of 18? Of 21? Of 23?

22. What is $\frac{1}{10}$ of 3? Of 5? Of 6? Of 7? Of 11? Of 19? Of 20?

23. What is $\frac{1}{9}$ of 4? Of 7? Of 9? Of 10? Of 13? Of 15? Of 16?

24. What is $\frac{2}{8}$ of 7? Of 8? Of 9? Of 10? Of 11? Of 12? Of 14?

25. What is $\frac{3}{4}$ of 2? Of 7? Of 9? Of 13? Of 14? Of 18? Of 19?

26. How many are $\frac{2}{7}$ of 5? Of 6? Of 10? Of 11? Of 12? Of 31?

27. How many are $\frac{4}{9}$ of 2? Of 3? Of 6? Of 10? Of 13? Of 20?

28. How many are $\frac{5}{11}$ of 2? Of 4? Of 7? Of 9? Of 10? Of 12?

29. How many are $1\frac{2}{5}$ times 3? Times 5? Times 7? Times 9? Times 20?

30. How many are $2\frac{3}{6}$ times 2? Times 8? Times 9? Times 10? Times 15?

31. 6 is $\frac{4}{5}$ of what number? $\frac{3}{5}$? $\frac{5}{6}$? $\frac{9}{10}$?

32. 8 is $2\frac{1}{4}$ times what number? $1\frac{3}{5}$? $2\frac{2}{7}$?

33. What number added to $\frac{2}{3}$ of 31 will make 25?

34. After spending a portion of my money, I had $20 left, which was just $\frac{2}{13}$ of what I had at first; how much had I at first?

35. A gentleman is 83 years old, and his son is $\frac{3}{5}$ as old; how old is his son?

36. What cost 2 dozen eggs, at the rate of 3 for 7 cents?

37.   What cost 9 bushels of quinces, at the rate of 4 bushels for $7?

38.   If 5 oranges cost 28 cents, what will 11 oranges cost?

39.   If $31 is $\frac{3}{4}$ of what money William has, how much has he?

40.   A boy having 66 cents gave $\frac{8}{11}$ of them to his mother; how many did he give her?

41.   Sold a horse for $64, which was $1\frac{1}{7}$ times what he cost me; what did he cost me?

42. $\frac{1}{2}$ of Edward's money is 5 times Henry's, and Henry has $11; how much has Edward?

43. $\frac{2}{3}$ of 24 is $\frac{4}{15}$ of the number of books which James owns; how many does he own?

44. $\frac{4}{7}$ of 28 is $\frac{8}{9}$ of what number?

45. $\frac{1}{8}$ of 16 is $\frac{1}{4}$ of what number?

46. $\frac{5}{6}$ of 42 is $\frac{2}{3}$ of what number?

47. $\frac{3}{4}$ of 48 is $\frac{6}{9}$ of what number?

48. $\frac{4}{11}$ of 22 is $\frac{8}{10}$ of what number?

49. $\frac{7}{8}$ of 32 is $\frac{8}{7}$ of what number?

50. $\frac{6}{7}$ of 56 is $1\frac{2}{4}$ times what number?

51. $\frac{4}{5}$ of 75 is $2\frac{2}{9}$ times what number?

52.   Bought a cow and a horse; the cost of the cow was $32, and that of the horse was 2 times $\frac{8}{9}$ of the cost of the cow; what was the cost of the horse?

53.   If 3 quarts of cranberries, at 80 cents a peck, will pay for 4 pounds of rice, how much is the rice a pound?

54. $\frac{5}{6}$ of 48 is how many times $\frac{1}{7}$ of 35?

55. $\frac{7}{9}$ of 72 is how many times $\frac{3}{4}$ of 12?

56.   What number taken from $\frac{3}{4}$ of 26 will leave $16\frac{1}{8}$?

57.   If $\frac{3}{4}$ of a pound of raisins are worth 8 cents, how much would 5 pounds be worth?

LESSON XXXIV.

1. When 3 pounds of coffee are worth $\frac{9}{10}$ of a dollar, what is 1 pound worth?

ANALYSIS. — *If 3 pounds of coffee are worth* $\$\frac{9}{10}$, 1 *pound is worth* $\frac{1}{3}$ *of* $\$\frac{9}{10}$, *which is* $\$\frac{3}{10}$.

2. If 1 rod of land is worth $\$1\frac{2}{3}$, how much is $\frac{1}{4}$ of a rod worth?

3. If 9 pounds of tea cost $\$5\frac{2}{3}$, what does it cost a pound?

NOTE. — Reduce $5\frac{2}{3}$ to an improper fraction.

4. If a man can reap a field in $2\frac{2}{3}$ days, in how long a time will he reap $\frac{1}{10}$ of the field?

5. John had $\frac{1}{5}$ of a dollar, and gave $\frac{1}{2}$ of it to James; what part of a dollar did James receive?

ANALYSIS. $\$\frac{1}{5}$ *is equal to* $\$\frac{2}{10}$; *and since John gave James* $\frac{1}{2}$ *of* $\$\frac{1}{5}$, *he gave him* $\frac{1}{2}$ *of* $\$\frac{2}{10}$, *or* $\$\frac{1}{10}$.

NOTE. — We reduce the fraction $\$\frac{1}{5}$ to higher terms by multiplying both numerator and denominator by 2, and we are then able to divide the numerator by 2. The same result could more easily be obtained by merely multiplying the denominator by 2, that is, by multiplying the denominators together; thus, $\frac{1}{2}$ of $\$\frac{1}{5} = \$\frac{1}{10}$. *Multiplying the denominator of a fraction by any number produces the same effect upon the value of the fraction as dividing its numerator by the same number.*

6. William divided $\frac{1}{4}$ of a melon equally among 3 persons; what part of a whole melon did he give to each? (See Appendix.)

7. A gentleman owned $\frac{1}{6}$ of a ship; if he should sell $\frac{1}{4}$ of his share, what part of the ship would he sell?

8. What is $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{4}$?

9. Mary gathered $\frac{1}{2}$ of a box of strawberries, and divided them equally among 4 persons; what part of a box did she give to each?

10. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{6}$ of $\frac{1}{3}$?

11. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{4}$?

12. What is $\frac{1}{8}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{6}$ of $\frac{1}{6}$? $\frac{1}{7}$ of $\frac{1}{7}$?

13. What is $\frac{1}{5}$ of $\frac{4}{9}$?

ANALYSIS. $\frac{1}{5}$ of $\frac{1}{9}$ is $\frac{1}{45}$, and $\frac{1}{5}$ of $\frac{4}{9}$ is 4 times $\frac{1}{45}$, or $\frac{4}{45}$.

14. What is $\frac{1}{10}$ of $\frac{4}{5}$? $\frac{1}{7}$ of $\frac{3}{8}$? $\frac{1}{8}$ of $\frac{3}{7}$? $\frac{1}{8}$ of $\frac{5}{6}$?

15. What is $\frac{1}{6}$ of $\frac{9}{10}$? $\frac{1}{7}$ of $\frac{14}{21}$? $\frac{1}{10}$ of $\frac{5}{17}$? $\frac{1}{11}$ of $\frac{8}{11}$? $\frac{1}{12}$ of $\frac{6}{7}$?

16. A gentleman, owning $\frac{3}{4}$ of a farm, sold $\frac{1}{3}$ of his part; what part of the whole farm did he sell?

17. If 6 pounds of cheese cost $\frac{7}{8}$ of a dollar, what does 1 pound cost?

18. If a barrel of potatoes cost $\$ 2\frac{1}{2}$, what will $\frac{1}{8}$ of a barrel cost?

19. Bought 5 barrels of flour for $\$ 34\frac{5}{6}$; what was the cost of 1 barrel?

ANALYSIS. — *Since 5 barrels of flour cost* $\$ 34\frac{5}{6}$, 1 *barrel cost* $\frac{1}{5}$ *of* $\$ 34\frac{5}{6}$; $\frac{1}{5}$ *of* $\$ 34$ *is* $\$ 6$, *with a remainder of* $\$ 4$; $\$ 4\frac{5}{6} = \$ \frac{29}{6}$, *and* $\frac{1}{5}$ *of* $\$ \frac{29}{6}$ *is* $\$ \frac{29}{30}$. *Therefore the cost of 1 barrel was* $\$ 6\frac{29}{30}$.

20. If 4 pounds of sugar cost $33\frac{1}{3}$ cents, what cost 1 pound? What is $\frac{1}{4}$ of $33\frac{1}{3}$?

21. Sold 9 barrels of apples for $\$ 18\frac{3}{4}$; what cost 1 barrel? What is $\frac{1}{9}$ of $18\frac{3}{4}$?

22. What is $\frac{1}{8}$ of $4\frac{1}{2}$? $\frac{1}{4}$ of $3\frac{1}{4}$? $\frac{1}{6}$ of $5\frac{1}{4}$? $\frac{1}{5}$ of $10\frac{1}{2}$?

23. What is $\frac{1}{7}$ of $11\frac{1}{3}$? $\frac{1}{8}$ of $7\frac{1}{5}$? $\frac{1}{10}$ of $16\frac{3}{4}$? $\frac{1}{9}$ of $11\frac{1}{6}$?

24. What part of an acre is $\frac{4}{5}$ of $\frac{7}{11}$ of an acre?

ANALYSIS. $\frac{1}{5}$ *of* $\frac{7}{11}$ *of an acre is* $\frac{7}{55}$ *of an acre, and* $\frac{4}{5}$ *of* $\frac{7}{11}$ *are 4 times* $\frac{7}{55}$, *or* $\frac{28}{55}$, *of an acre.*

25. What part of a ship is $\frac{3}{4}$ of $\frac{2}{3}$ of it?

26. If you should have $\frac{4}{7}$ of a barrel of apples,

and sell ⅝ of them, what part of a barrel would you sell?

27. What is ½ of ⅔? ⅔ of ⅚? ⅔ of ⅕? ⅜ of ⅚?

28. What is ⅘ of ⅔? ¾ of 1⅓? ⅞ of ⁹⁄₁₀? ⅔ of ¾? ⁴⁄₁₀ of ⅚?

29. What is ⁷⁄₁₁ of ⅕? ⅝ of ⅘? ⅔ of 2⅚? ⅘ of 1⁹⁄₁₁? ⅜ of 2⅔?

30. What cost ⅚ of a bushel of corn, at ⅞ of a dollar a bushel? (Ex. 11, Les. XXXIII.)

31. If 3 pounds of butter cost ¾ of a dollar, what cost 10 pounds?

32. If 4 men can do a piece of work in 16⅓ days, how long will it take 3 men to do the same?

33. If ¾ of a rod of land is worth ⅞ of a dollar, what is a rod worth?

ANALYSIS. — *If ¾ of a rod of land are worth $⅞, ¼ of a rod is worth ⅓ of $⅞, or $ ₇⁄₂₄, and ¼, or 1 rod, is worth 4 times $ ₇⁄₂₄, which are $⅞, or $ 1⅙.*

34. When 2½ pounds of coffee can be bought for $ ¹¹⁄₁₂, what is it worth a pound?

NOTE. — The mixed number must be reduced to an improper fraction.

35. 2½ is ¾ of what number? ⅚? ⁴⁄₇? ¹⁰⁄₁₂?

36. ⅚ is 3⅓ times what number? 1⅘? 1⅓?

37. If 7 bushels of potatoes cost $ 8⅔, how much are they a bushel?

38. Bought 6 yards of cloth for $ 12¾; what was the cost of one yard?

39. If 7 yards of broadcloth cost $ 37⁸⁄₁₀, what will 8 yards cost?

40. If Joseph can walk 6⅔ miles in 2 hours, how many miles can he walk in 8 hours?

41. If ⅔ of a barrel of beef cost $ 3¾, what will ⅖ of a barrel cost?

42. What will $1\frac{1}{4}$ yards of cloth cost, if $\frac{4}{5}$ of a yard cost $\frac{3}{8}$ of a dollar?

43. John, who is $16\frac{1}{4}$ years old, is $1\frac{1}{2}$ times as old as George; what is George's age?

44. Gave $\frac{2}{5}$ of $\$4\frac{1}{4}$ for a hat; what did the hat cost?

45. If a pine-apple costs $9\frac{3}{8}$ cents, what will $1\frac{1}{5}$ pine-apples cost?

46. John can reap an acre in $12\frac{1}{4}$ hours, and George as much in $\frac{3}{4}$ of that time; how long will it take George to reap an acre?

47. If a ship sails $3\frac{1}{4}$ miles in $\frac{4}{5}$ of an hour, how far will it sail in 5 hours?

48. If a bushel of apples will buy $\frac{5}{8}$ of a bushel of peaches, what quantity of peaches can be bought for $\frac{4}{5}$ of a bushel of apples?

49. What cost 3 quarts of beans, if 1 peck cost $\frac{7}{8}$ of a dollar?

50. If 6 pounds of sugar cost $\frac{4}{5}$ of a dollar, what cost 5 pounds?

51. If 3 tons of hay cost $\$31\frac{1}{2}$, what cost $\frac{3}{4}$ of a ton?

52. If a man earn $\$8\frac{4}{7}$ in 6 days, how much can he earn in $\frac{1}{4}$ of $\frac{4}{7}$ of a day?

53. When $\frac{2}{5}$ of $\$6\frac{1}{2}$ will buy a yard of broadcloth, how much will $\frac{3}{11}$ of a yard cost?

54. Sold $\frac{4}{7}$ of $5\frac{1}{2}$ pounds of coffee, at $1\frac{1}{2}$ dimes a pound; how much did it come to?

55. Andrew is 20 years old, and his age exceeds by 5 years $\frac{3}{4}$ of $\frac{5}{7}$ of Walter's age; how old is Walter?

56. If $2\frac{1}{8}$ tons of hay are worth 2 times $11\frac{1}{8}$ dollars, how many dollars are $2\frac{1}{4}$ tons worth?

LESSON XXXV.

1. At $\frac{4}{9}$ of a dollar a yard, how many yards of cloth can be bought for $\$3\frac{1}{9}$?

·FIRST ANALYSIS. $\$3\frac{1}{9} = \$\frac{28}{9}$. *Since* $\$\frac{4}{9}$ *will buy 1 yard of cloth,* $\$\frac{28}{9}$ *will buy as many yards as 4 ninths of a dollar are contained times in 28 ninths of a dollar, which are* 7.

SECOND ANALYSIS. — *Since* $\$\frac{4}{9}$ *will buy 1 yard,* $\$\frac{1}{9}$ *will buy* $\frac{1}{4}$ *of a yard, and* $\$\frac{28}{9}$ *will buy 28 times* $\frac{1}{4}$ *of a yard, which are* $\frac{28}{4}$ *of a yard, or* 7 *yards.*

2. At $\frac{3}{4}$ of a dollar a day, how long will it take a man to earn 9 dollars?

3. How many chairs, at $\$5\frac{1}{2}$ apiece, can be bought for $\$38\frac{1}{2}$?

4. How many bushels of wheat, at $\$2\frac{3}{8}$ a bushel, can be bought for $\$19$?

5. If a man can build $4\frac{2}{5}$ rods of wall in a day, how many days will it take him to build $13\frac{1}{5}$ rods?

6. How many yards of cloth, at $\frac{1}{4}$ of a dollar a yard, can be bought for 2 dollars? How many times $\frac{1}{4}$ in 2?

7. How many pounds of sugar can be bought for 50 cents, when it is $8\frac{1}{3}$ cents a pound?

8. Susan distributed 4 pears among some school-mates, giving to each $\frac{2}{5}$ of a pear; how many schoolmates were there?

9. If you spend $\frac{3}{4}$ of a dollar a week, how long will you be in spending 6 dollars?

10. How many times $\frac{2}{5}$ in 4? In 6?

11. How many times $2\frac{1}{3}$ in 12?

NOTE. $2\frac{1}{3} = \frac{7}{3}$, and $12 = \frac{36}{3}$; $\frac{7}{3}$ is contained in $\frac{36}{3}$ as many times as 7 in 36, or $5\frac{1}{7}$ times.

12. How many times $2\frac{1}{3}$ in 9?

13. How many times 8 in $9\frac{1}{4}$? In $10\frac{1}{5}$?

14. How many times 4 in $11\frac{1}{5}$? In $12\frac{1}{4}$?

5

15. How many times $\frac{2}{3}$ are $13\frac{1}{3}$?

16. How many times $\frac{5}{6}$ are $2\frac{3}{8}$?

17. How many times $\frac{6}{7}$ are $5\frac{6}{7}$?

18. How many times $1\frac{1}{2}$ are $6\frac{1}{2}$?

19. How many times $1\frac{1}{3}$ are $6\frac{2}{3}$?

20. How many times $2\frac{1}{6}$ are $8\frac{4}{6}$?

21. How many times $3\frac{1}{8}$ are $6\frac{2}{8}$?

22. $4\frac{3}{7}$ are how many times $\frac{1}{7}$ of 5?

23. $6\frac{2}{9}$ are how many times $\frac{1}{9}$ of 7?

24. $4\frac{4}{6}$ are how many times $\frac{1}{2}$ of $\frac{7}{8}$?

25. $9\frac{1}{2}$ are how many times $2\frac{1}{2}$?

26. $16\frac{2}{3}$ are how many times $\frac{1}{4}$ of $33\frac{1}{3}$?

27. $10\frac{2}{5}$ are how many times 2?

28. When eggs are $\frac{1}{6}$ of a dollar a dozen, how many dozen must be given for 16 pounds of coffee, at $\frac{1}{8}$ of a dollar a pound?

29. If a lamb costs $\$2\frac{2}{5}$, how many lambs can be bought for $\$10\frac{2}{5}$?

30. If a teacher distributed in rewards among his pupils $\$7\frac{1}{5}$, by giving $\frac{4}{5}$ of a dollar to each, how many pupils did he reward?

31. How many lambs, at $\$2\frac{1}{4}$ apiece, may be had for 10 calves, at $\$6\frac{3}{4}$ apiece?

32. How many bushels of apples, at $\frac{5}{16}$ of a dollar a bushel, will pay for 15 yards of calico, at $\frac{3}{16}$ of a dollar a yard?

33. If $\$1\frac{1}{3}$ will pay 1 man for a day's work, how many men will $\$6\frac{3}{4}$ pay?

34. How many pens, at $1\frac{1}{2}$ cents apiece, can be bought for 12 cents?

35. If 1 man consumes $1\frac{3}{8}$ pounds of meat in one day, how many men will consume $15\frac{1}{8}$ pounds in the same time?

36. At $\$2\frac{1}{4}$ a pair, how many pairs of shoes can be bought for $\$15\frac{3}{4}$?

37. How many bushels of potatoes, at $\frac{3}{16}$ of a dollar a peck, must be given for 10 bushels of wheat, at $\$2\frac{4}{16}$ a bushel?

38. If a man can mow $1\frac{1}{2}$ acres in 1 day, how long will it take 6 men to mow $14\frac{2}{5}$ acres?

LESSON XXXVI.

1. If a man walks $2\frac{1}{2}$ miles an hour, in how many hours will he walk $8\frac{1}{3}$ miles?

ANALYSIS. — *At $2\frac{1}{2}$ miles an hour, he will walk $8\frac{1}{3}$ miles in as many hours as $2\frac{1}{2}$ miles are contained times in $8\frac{1}{3}$ miles. $2\frac{1}{2} = \frac{5}{2} = \frac{15}{6}$, $8\frac{1}{3} = \frac{25}{3} = \frac{50}{6}$; and $\frac{15}{6}$ are contained in $\frac{50}{6}$, $3\frac{5}{15}$, or $3\frac{1}{3}$, times.*

SECOND METHOD OF DIVIDING. 1 *is contained in $\frac{2}{3}$, $2\frac{1}{2}$ times; $\frac{1}{2}$ is contained in $\frac{2}{3}$, 2 times $\frac{2}{3}$, or $\frac{4}{6}$ times; and $\frac{5}{2}$ are contained in $\frac{2}{3}$, $\frac{1}{5}$ of $\frac{4}{6}$ times, which is $\frac{4}{10}$, or $3\frac{1}{3}$ times.*

2. At $\frac{1}{4}$ of a dollar apiece, how many tickets can be purchased for $\frac{1}{2}$ of a dollar?

3. At $\frac{1}{12}$ of a dollar a pound, how many pounds of starch can be purchased for $\frac{3}{4}$ of a dollar?

4. If a horse can trot $\frac{1}{8}$ of a mile in a minute, how long will it take him to trot $\frac{5}{8}$ of a mile?

5. If 1 man can do a piece of work in $5\frac{1}{4}$ days, how many men can do it in $2\frac{3}{4}$ days?

6. How many men will be required to reap $2\frac{1}{8}$ acres, while 1 man is reaping $1\frac{1}{4}$ acres?

7. When $\$1$ will buy $2\frac{1}{2}$ gallons of molasses, how many dollars must be paid for $5\frac{1}{4}$ gallons?

8. At $\frac{5}{8}$ of a dollar a day, in how many days will a man earn $\$6\frac{1}{4}$?

9. At $\frac{3}{5}$ of a dollar a day, in what part of a day will a man earn $\frac{1}{2}$ of a dollar?

FIRST ANALYSIS. — *At $ $\frac{3}{5}$ a day, a man will earn $ $\frac{1}{5}$, or $ $\frac{3}{5}$, in the same part of a day, that $ $\frac{3}{5}$ is of $ $\frac{3}{5}$, or that 3 is of 5, which is $\frac{3}{5}$. Therefore he will earn $ $\frac{1}{5}$ in $\frac{3}{5}$ of a day.*

SECOND ANALYSIS. — *If a man earn $ $\frac{3}{5}$ in 1 day, he will earn $ $\frac{1}{5}$ in $\frac{1}{5}$ of a day, and $ $\frac{3}{5}$ in 3 times $\frac{1}{5}$, or $\frac{3}{5}$, of a day.*

10. At $\frac{5}{6}$ of a dollar a day, in what part of a day will a man earn $\frac{3}{4}$ of a dollar?

11. If a man can do a piece of work in $\frac{7}{8}$ of a day, what part of the work can he do in $\frac{3}{4}$ of a day?

12. How many times is $\frac{2}{3}$ contained in $\frac{5}{6}$?

13. How many times is $\frac{2}{5}$ contained in $1\frac{7}{6}$?

14. If a horse eats $\frac{1}{6}$ of a ton of hay in 1 week, how many weeks will he be in eating $\frac{8}{9}$ of a·ton?

15. How many times is $\frac{3}{4}$ contained in $\frac{9}{16}$?

16. How many pounds of tea, at $\frac{3}{8}$ of a dollar a pound, can be bought for $\frac{9}{12}$ of a dollar?

17. How many times is $\frac{4}{5}$ contained in $2\frac{1}{5}$?

18. How many times is $\frac{2}{7}$ contained in $\frac{9}{14}$?

19. How many times is $3\frac{1}{5}$ contained in $6\frac{4}{10}$?

20. How many times is $\frac{4}{5}$ contained in $11\frac{1}{5}$?

21. How many times is $\frac{2}{9}$ contained in $\frac{3}{4}$?

22. How many times is $\frac{1}{4}$ contained in $\frac{5}{12}$?

23. How many times is $6\frac{1}{4}$ contained in $12\frac{5}{8}$?

24. If a lamb cost $ $2\frac{1}{6}$, how many lambs can be bought for $ $8\frac{2}{3}$?

25. At $\frac{3}{8}$ of a dollar a yard, how many yards of cloth can be bought for $ $4\frac{1}{2}$?

26. If 1 man can do a piece of work in $6\frac{2}{3}$ days, how many men must be employed to do it in $2\frac{2}{5}$ days?

27. How many bushels of apples, at $\frac{3}{8}$ of a dollar

a bushel, must be given for 7 bushels of potatoes, at $\frac{3}{4}$ of a dollar a bushel?

28. If a man can walk $\frac{6}{10}$ of a mile in $\frac{1}{6}$ of an hour, in how many hours can he walk $8\frac{1}{4}$ miles?

29. How many pounds of sugar, at $\frac{1}{12}$ of a dollar a pound, can be bought for $\frac{3}{4}$ of a bushel of cranberries, at $ 3\frac{1}{2}$ a bushel?

30. When 1 yard of cloth costs $ 1\frac{1}{6}$, how much cloth can be bought for $ 1\frac{1}{4}$?

31. A farmer, having sold a load of wood for $ 7\frac{1}{2}$, spent $\frac{2}{3}$ of the money for tea, at $\frac{3}{4}$ of a dollar a pound, and the balance for coffee, at $\frac{1}{6}$ of a dollar a pound; how many pounds of each did he obtain?

32. 8 dozen of eggs, at $ \frac{1}{4}$ a dozen, will pay for how many yards of cloth, at $ \frac{7}{10}$ a yard?

33. How many times is $\frac{1}{6}$ of $\frac{3}{4}$ contained in $\frac{4}{6}$ of $2\frac{1}{5}$? In $\frac{5}{6}$ of $\frac{9}{10}$?

34. When oats are $\frac{5}{16}$ of a dollar a bushel, and corn $\frac{7}{8}$ of a dollar a bushel, how many bushels of oats must be given for 5 bushels of corn?

LESSON XXXVII.

RATIO is the relation, in respect to magnitude or value, which one quantity or number bears to another of the same kind. It is expressed by dividing one quantity by the other. Thus, the ratio of 6 books to 3 books is 2, because 6 books are twice 3 books; the ratio of 3 to 6 is $\frac{1}{2}$, because 3 is $\frac{1}{2}$ of 6; the ratio of 7 to 3 is $\frac{7}{3}$, because 7 is $\frac{7}{3}$ of 3; and the ratio of 3 to $\frac{7}{8}$ is $\frac{24}{7}$, because 3 is $\frac{24}{7}$ of $\frac{7}{8}$.

1. What part of 3 is 1? Of 3 is 2?

NOTE. — See Exs. 2, 20, Lesson XXVI.

2. What part of 5 is 1? Of 5 is 3?

3. What part of 9 is 1? Of 9 is 5?

4. What part of 7 is 3? Of 8 is 5?
5. What part of 11 is 5? Of 13 is 6?
6. What· part of 6 is 7? Of 9 is 11?
7. What part of 13 is 14? Of 14 is 17?
8. What part of 2 is $\frac{1}{4}$? Of $\frac{3}{4}$ is $\frac{2}{7}$?

ANALYSIS. $2 = \frac{8}{4}$, and $\frac{1}{4}$ is the same part of $\frac{8}{4}$ that 1 is of 8, or $\frac{1}{8}$.

$\frac{3}{4} = \frac{15}{20}$, $\frac{2}{7} = \frac{8}{20}$, and $\frac{8}{20}$ is the same part of $\frac{15}{20}$ that 8 is of 15, or $\frac{8}{15}$.

9. What part of 5 is $\frac{1}{6}$? Of 7 is $\frac{4}{6}$?
10. What part of 9 is $\frac{1}{4}$? Of 10 is $\frac{5}{6}$?
11. What part of $\frac{7}{8}$ is 3? Of $\frac{5}{6}$ is 5?
12. What part of $\frac{6}{7}$ is $\frac{1}{7}$? Of $\frac{5}{9}$ is $\frac{1}{5}$?
13. What part of $\frac{9}{10}$ is $\frac{1}{6}$? Of $\frac{8}{11}$ is $\frac{1}{4}$?
14. What part of $\frac{8}{9}$ is $\frac{3}{4}$? Of $\frac{5}{6}$ is $\frac{2}{3}$?
15. What part of $\frac{1}{4}$ is $\frac{4}{6}$? Of $\frac{2}{3}$ is $\frac{6}{11}$?
16. What part of 1 peck is 5 quarts?
17. What part of 1 pound is 9 shillings?
18. What part of 1 quarter is $14\frac{1}{2}$ pounds?

NOTE. — Reduce both to half-pounds.

19. What part of 2 dimes is 13 cents?
20. What part of 1 week 4 days is 5 days?
21. What part of 5 bushels is 3 pecks 6 quarts?
22. What part of 10 pounds is 4 pounds 12 ounces?
23. What part of 2 hogsheads is 1 hogshead 17 gallons?
24. What part of 3 yards 3 quarters is 3 quarters 3 nails?
25. What part of 3 leagues 2 miles is 1 mile 3 furlongs?
26. What part of 1 acre 2 roods is 3 roods 15 square rods?
27. What part of a mile is 3 furlongs 15 rods?

28. What part of 8 yards is 5 ells English 3 quarters?

29. What part of a pint is $\frac{1}{12}$ of a gallon?

ANALYSIS. — *Since 1 gallon = 4 quarts, $\frac{1}{12}$ of a gallon = $\frac{1}{12}$ of 4 quarts, which is $\frac{4}{12}$, or $\frac{1}{3}$, of a quart; and since 1 quart = 2 pints, $\frac{1}{3}$ of a quart = $\frac{1}{3}$ of 2 pints, or $\frac{2}{3}$ of a pint.*

30. What part of an inch is $\frac{1}{48}$ of a yard?

31. What part of a pennyweight is $\frac{1}{28}$ of an ounce?

32. What part of a rod is $\frac{1}{400}$ of a mile?

33. What part of a day is $\frac{2}{15}$ of a week?

34. What part of a gill is $\frac{2}{65}$ of a gallon?

35. What part of a shilling is $\frac{1}{6}$ of a farthing?

ANALYSIS. — *Since 4 farthings = 1 penny, 1 farthing = $\frac{1}{4}$ of a penny, and $\frac{1}{6}$ of a farthing = $\frac{1}{6}$ of $\frac{1}{4}$ of a penny, or $\frac{1}{24}$ of a penny; and since 12 pence = 1 shilling, 1 penny = $\frac{1}{12}$ of a shilling, and $\frac{1}{6}$ of a penny = $\frac{1}{6}$ of $\frac{1}{12}$ of a shilling, or $\frac{1}{60}$ of a shilling.*

36. What part of a week is $\frac{2}{3}$ of a day?

37. What part of a yard is $\frac{3}{10}$ of an inch?

38. What part of a ton is $\frac{1}{4}$ of a pound?

39. What part of a bushel is $\frac{2}{3}$ of a pint?

40. What part of a pound is $\frac{1}{2}$ of a pennyweight?

41. What is the value of $\frac{1}{8}$ of a hogshead, in gallons?

42. What is the value of $\frac{3}{8}$ of a pound, in shillings and pence?

ANALYSIS. — *Since 1 pound = 20 shillings, $\frac{3}{8}$ of a pound = $\frac{3}{8}$ of 20 shillings, which are $7\frac{1}{2}$ shillings; and since 1 shilling = 12 pence, $\frac{1}{2}$ of a shilling = $\frac{1}{2}$ of 12 pence, or 6 pence. Therefore $\frac{3}{8}$ of a pound = 7 shillings 6 pence.*

43. What is the value of $\frac{3}{8}$ of a pound Troy?

44. What is the value of $\frac{5}{6}$ of a yard?

45. What is the value of $\frac{3}{4}$ of a hogshead?

46. What is the value of $\frac{4}{7}$ of a bushel?

47. What is the value of $\frac{3}{16}$ of a ton weight?

48. What is the value of $\frac{3}{11}$ of a week?

LESSON XXXVIII.

1.   A market-woman sold some butter, eggs, and milk; for the eggs and milk she received $ 4, which was ⅔ of what she received for the butter; how much did she receive for the butter?

2.   Sold ⅝ of an acre of land for $ 24; at how much an acre was it sold?

3.   If James can run 60 rods in a minute, and John ⅔ as far, how long will it take John to run 15⅓ rods?

4.   28 is ⅞ of a number; what is that number?

5.   36 is ₆/₁₁ of a number; what is that number?

6.   If ⅖ of a firkin of butter cost $ 8, what will ¾ of a firkin cost?

7.   ½ of ¼ of 24 is ⅔ of what number?

8.   ⅓ of ⅖ of 15 is ⅔ of what number?

9.   If a bushel of potatoes is ⅔ of a barrel, and cost $ 1⅓, how much will 1 barrel cost?

10.   A and B own some wood together; A's share is ⅖; it is sold, and A receives as ½ of his part of the proceeds $ 24; how much did the wood sell for? How much was B's part of the proceeds?

11.   A teacher, being asked how many scholars he had, answered that 20 of them were girls, and ⅖ of them and 4 more were boys; how many boys were there? How many scholars in all?

ANALYSIS. — *As the number of scholars is ⅖ of itself, 20 scholars + 4 scholars must be the remaining ⅗. If ⅗ of the number is 24, ⅕ of the number is ⅓ of 24, or 8, and ⅗, or the whole number of scholars, is 5 times 8, or 40. The number of boys must be the difference between 40 and 20, which is 20.*

12.   14 is ⅖ of ⅞ of what number?

13.   18 is ¾ of ⅗ of what number?

14.   John Jones sold a horse for $60, which was 2 times $\frac{2}{5}$ of what he gave for it; how much did he gain by the sale?

15.   Bought a piece of land for $64, and sold $\frac{1}{4}$ of $\frac{4}{7}$ of it for $28; how much will be gained if the rest be sold at the same rate?

16. $\frac{4}{5}$ of a pole is above ground, and 8 feet is $\frac{1}{2}$ of the part in the ground; what is the length of the pole?

17.   George spent 6 elevenths of his money for a suit of clothes; he then paid $2 for a hat, which was just 1 fifth of all he had left; how many dollars had he at first?

18.   35 is $\frac{5}{6}$ of $\frac{7}{8}$ of what number?

19.   A pole stands 3 fifths in the water, 1 half of the remainder in the mud, and 4 feet above the water; what is the length of the pole?

20.   If $\frac{2}{3}$ of a dozen of eggs cost $\frac{1}{6}$ of a dollar, how many dozen can be bought for $1?

21.   36 is $\frac{3}{4}$ of how many times 8?

22.   52 is $\frac{4}{5}$ of how many times 13?

23.   John Doe and Richard Roe enter into a speculation together, with a certain capital, $\frac{4}{5}$ of $\frac{2}{3}$ of which was contributed by John, and the remainder by Richard; on dividing their gains, John received $60 as his share; how much was Richard's share?

24.   Henry spent $\frac{1}{4}$ of his money for pencils, $\frac{2}{3}$ for a slate, and had 4 cents left; how much money had he, and how many pencils did he buy, at 2 cents apiece?

25.   30 is $\frac{5}{6}$ of how many times 9?

26.   64 is $\frac{8}{9}$ of how many times 12?

27.   If $\frac{2}{5}$ of Robert's age has passed since he was 21 years old, how old is he?

28. $\frac{4}{5}$ of $\frac{4}{8}$ is how many times $\frac{1}{7}$?

29. Susan is 25 years old, and Lydia is only $6\frac{1}{4}$ years; how many times is Susan as old as Lydia?

30. A man sold $\frac{4}{7}$ of a barrel of cider, and $\frac{1}{4}$ of what was left was worth $1\frac{1}{5}$; what was the whole barrel worth?

31. What cost $\frac{1}{5}$ of a hogshead of molasses, at $\frac{4}{5}$ of a dollar a gallon?

32. At $\frac{1}{12}$ of a dollar a quart, what part of a bushel of walnuts can be bought for $2\frac{1}{4}$?

33. Allowing 10 working hours to a day, in what part of a week, consisting of 6 working days, can a man earn $7, if his pay be at the rate of $\frac{1}{5}$ of a dollar an hour?

34. If $1\frac{1}{4}$ pecks be added to $\frac{3}{16}$ of a bushel, what part of a bushel will there then be?

35. If from $\frac{5}{8}$ of a hundred-weight there be taken $\frac{1}{4}$ of a quarter, what part of a hundred-weight will then remain?

36. A young man lost $\frac{1}{4}$ of his capital in trade, but afterwards gained $100, which made his capital $1000; how much money did he lose?

37. 60 is $\frac{5}{7}$ of how many times $\frac{1}{8}$ of 32?

38. 84 is $\frac{7}{8}$ of how many times $\frac{1}{2}$ of 22?

39. 18 is $\frac{3}{7}$ of how many fifths of 35?

40. 20 is $\frac{5}{9}$ of how many sevenths of 42?

41. If $3\frac{2}{8}$ pounds of butter last a family 1 week, how long will $25\frac{2}{8}$ pounds last the same family?

42. How many times is $1\frac{4}{9}$ contained in $11\frac{4}{9}$?

43. How many times $33\frac{1}{3}$ are 6 times $16\frac{2}{3}$?

44. How many times 3 times $2\frac{1}{5}$ are $10\frac{2}{5}$?

45. A watch-chain cost $48, and $\frac{3}{4}$ of the cost of the chain was $\frac{2}{5}$ of the cost of the watch; what was the cost of the watch?

46. Joseph deposited $\frac{1}{4}$ of his money in the

savings-bank, paid ⅔ for tuition, and then found ¼ of the remainder was just $7; how much money had he in all?

47. A gentleman paid away a sum of money: ⅔ to James Brown, ¾ of ⅓ of the remainder, which was $15, to Peter Smith, and the balance to Edward Robinson; what was the whole amount, and how much did Brown and Robinson each receive?

LESSON XXXIX.

1. If 7 is ⅜ of some number, what is ¼ of the same number?

2. If 10 is ⅖ of some number, what is ⅝ of the same number?

3. If 12 is ⅖ of some number, what is ⅞ of the same number?

4. What number added to 2 times ⅓ of 37 will make the number 18?

5. What number added to 5 times ¾ of 30 will make the number 65?

6. What number taken from 4 times ⅝ of 19 will leave 50⅛?

7. What number taken from 8 times ¼ of 22 will leave 100?

8. What number is that to which if ⅕ of itself be added, the number will be 48?

NOTE. — The number is ⅕ of itself, and ⅕ added make ⅕₁ therefore 48 is ⅖ of the number.

9. What number is that to which if its half and fourth be added, the sum will be 100?

10. A lady, being asked how old she was, answered, that, if ⅓ and ⅛ of her age were added to her age, it would be 99 years; how old was she?

11. Paid $\frac{1}{5}$ of my money for pencils, $\frac{1}{4}$ for an account-book, and 16 cents for India-rubber, and had $\frac{1}{3}$ as much left as I had in the beginning; how much had I at first?

12. A father left his son a legacy, $\frac{1}{4}$ of which he spent in 6 months, and $\frac{3}{4}$ of the remainder lasted him 8 months longer, when he had only \$100 remaining; what sum did his father leave him?

NOTE. \$100 was $\frac{1}{4}$ of the remainder, and the remainder was $\frac{3}{4}$ of the original sum.

13. A and B made an even exchange of horses; by the trade A lost 24 dollars, which was $\frac{2}{7}$ of the value of the horse which he had at first; what was the value of each horse?

14. If one man can cut $1\frac{1}{2}$ cords of wood in a day, how long will it take 3 men to cut the same?

15. If 3 horses consume $\frac{3}{4}$ of a bushel of oats in 2 days, how many horses will consume $3\frac{1}{2}$ bushels in the same time?

16. $\frac{2}{9}$ of 36 is $\frac{4}{7}$ of how many tenths of 20?

NOTE. $\frac{2}{9}$ of 36 is 8, 8 is $\frac{4}{7}$ of 14, $\frac{1}{10}$ of 20 is 2, and 2 is contained in 14, 7 times. Then $\frac{2}{9}$ of 36 is $\frac{4}{7}$ of $\frac{7}{10}$ of 20.

17. $\frac{2}{4}$ of 44 is $\frac{3}{5}$ of how many sevenths of 35?

18. $\frac{4}{8}$ of 30 is $\frac{5}{9}$ of how many ninths of 54?

19. $\frac{4}{5}$ of 50 is $\frac{5}{8}$ of how many sixths of 54?

20. Simon bought 2 dozen of eggs, at 6 cents a dozen, 2 pounds of beef, at 12 cents, and spent $\frac{1}{5}$ of what he had in the beginning for vegetables, and $\frac{1}{4}$ for a lobster, and had $1\frac{1}{5}$ left; how much had he at first?

21. If a man, by selling a cart for $1\frac{2}{3}$ times its cost, gains \$11, what was the cost of the cart?

22. Robert receives $\frac{3}{4}$ as much money on January 1st as he does on January 2d; he has in the whole $\frac{4}{5}$ of the amount required to pay his debts, and

after paying out all his money, he still owes 28 dollars; how much does he receive January 1st, and how much January 2d?

NOTE.—First find how much he received on both days.

23. Paid out $\frac{2}{5}$ of the money I had, and then borrowed $\frac{1}{2}$ as much as I had left; I had then 6 cents less than I had in the beginning; how much had I at first?

24. $\frac{4}{5}$ of 15 is how many eighths of 32?

25. $\frac{7}{8}$ of 24 is how many sixths of 18?

26. $\frac{5}{6}$ of 30 is $\frac{5}{9}$ of 9 times what number?

27. $\frac{6}{7}$ of 63 is $\frac{9}{10}$ of 12 times what number?

28. $\frac{11}{12}$ of 48 is $\frac{2}{3}$ of 5 times what number?

29. $\frac{5}{6}$ of 72 is $\frac{8}{11}$ of how many times 5?

30. $\frac{7}{10}$ of 80 is $\frac{6}{8}$ of how many times 12?

31. $\frac{5}{7}$ of 42 is $\frac{2}{3}$ of how many times $\frac{2}{3}$ of $13\frac{1}{2}$?

NOTE. $\frac{5}{7}$ of 42 is $\frac{2}{3}$ of 45; $\frac{2}{3}$ of $13\frac{1}{2}$ is 9; 45 are 5 times 9.

32. $\frac{3}{4}$ of 16 is $\frac{4}{5}$ of how many times $\frac{2}{5}$ of 15?

33. $\frac{4}{5}$ of 20 is $\frac{8}{9}$ of how many times $\frac{1}{3}$ of 54?

34. $\frac{2}{3}$ of 60 is $\frac{2}{3}$ of how many times $\frac{5}{8}$ of 40?

35. $\frac{2}{5}$ of $\frac{10}{12}$ of 27 is what part of 72?

NOTE. $\frac{2}{5}$ of $\frac{10}{12}$ of 27 is 9, and 9 is $\frac{1}{8}$ of 72.

36. $\frac{2}{3}$ of $\frac{9}{10}$ of 25 is what part of 30?

37. George, being asked his age, replied, if he were $\frac{1}{3}$ and $\frac{1}{6}$ older, he should be 40 years old; how old was he?

38. If a family of 6 persons will consume $2\frac{1}{4}$ barrels of flour in 6 months, what must be the number of persons in a family that will consume the same in $4\frac{1}{2}$ months?

39. $\frac{3}{4}$ of Henry's money is $\frac{4}{5}$ as much as Richard has, and twice Richard's money is $\frac{10}{7}$ as much as Charles has; if Charles has $42, how much each have Henry and Richard?

40. I have an orchard, in which ⅛ of the trees bear peaches, ⅔ bear apples, $\frac{1}{12}$ of them bear pears, 2 bear cherries, and 5 bear plums; how many trees are there in the orchard, and how many of each kind?

41. James spent ⅓ of his money for a coat, $\frac{1}{12}$ for a vest, and ¼ for a railroad ticket; he loses $18, borrows ⅛ as much as he had at the beginning, and has $8 left; how much had he at the. beginning?

42. A man, on his way to market, was met by another man, who said, " Good-morrow, sir, with your hundred geese." Said he, " I have not a hundred, but if I had ½ as many more and 2½ geese, I should have a hundred." How many had he ?

43. The head of a certain fish is 6 inches long, the tail is as long as the head and ½ of the body, and the body is 3½ times as long as the head; what is the length of the fish ?

44. A man has 2 horses and a saddle; the saddle, which is worth $24, when put upon one horse is worth 3 times as much as the horse, but when put upon the other horse, is worth ⅛ as much as the horse; what is the worth of each horse ? What is the worth of each horse, with the saddle ?

LESSON XL.

PER CENT. is a contraction of *per centum*, which means by the hundred. 1 per cent. means 1 of every hundred, or $\frac{1}{100}$; 3 per cent. is the same as $\frac{3}{100}$, and so on.

1. How much is 1 per cent. of $100 ?   Of $10 ? Of $1 ?

FIRST ANALYSIS.   1 *per cent., or* $\frac{1}{100}$, *of* $100 *is* $1; 1 *per cent. of* $10, *or* 100 *dimes, is* 1 *dime, or* 10 *cents; and* 1 *per cent. of* $1, *or* 100 *cents, is* 1 *cent.*

SECOND ANALYSIS. 1 *per cent., or* $\frac{1}{100}$, *of* $100 *is* $\frac{100}{100}$, *or* $1; 1 *per cent. of* $10 *is* $\frac{10}{100}$, *which is* $\frac{1}{10}$, *or* 10 *cents;* 1 *per cent. of* $1 *is* $\frac{1}{100}$, *or* 1 *cent.*

NOTE. — The first analysis corresponds with that of Ex. 21, Les. XVI., the second with that of Ex. 48, Les. XXVI., and both are used in Les. XXXIII. As 1 cent is $\frac{1}{100}$, 1 per cent. of any number of dollars must always be the same number of cents.

2. How much is 1 per cent. of $2? Of $4? Of $7? Of $13? Of $64?

3. How much is 1 per cent. of $200? Of $300? Of $500? Of $800?

4. How much is 1 per cent. of $150? Of $250? Of $450? Of $750?

5. How much is 1 per cent. of $15? Of $150? Of $175? Of $480?

6. How much is 2 per cent. of $100? Of $10? Of $1? Of 1 dime?

NOTE. — First find 1 per cent., as in Ex. 1, and 2 per cent. will be twice as much?

7. How much is 2 per cent. of $30? Of $50? Of $48? Of $64?

8. How much is 3 per cent. of $320? Of $460? Of $280?

NOTE. — If the number of cents or dimes is found to be inconveniently large, dollars and fractions of a dollar may be used, as whole numbers and fractions are used in the following examples.

9. How much is 5 per cent. of 320 bushels?

ANALYSIS. 1 *per cent. of* 320 *bushels is* $\frac{320}{100}$ *of a bushel, or* $3\frac{20}{100}$ *bushels, or* $3\frac{1}{5}$ *bushels; and* 5 *per cent. must be* 5 *times* $3\frac{1}{5}$ *bushels, or* 16 *bushels.*

10. How much is 4 per cent. of 225 tons?

11. How much is 6 per cent. of 100 pounds? Of 150 pounds? Of 360 pounds?

12. How much is 7 per cent. of 344 yards?

13.   How much is 20 per cent. of $280 ?   Of $560 ?   Of $600 ?

14.   How much is 15 per cent. of $1000 ?   Of $1500 ?   Of $2500 ?

15.   How much is 9 per cent. of 320 barrels ?

16.   A man divides his property among his four sons ; to the youngest he gives 15 per cent., to the next 20 per cent., to the second son 30 per cent., and the remainder to the eldest ; how much does· he give the eldest ?

NOTE. — The whole of anything is, of course, 100 per cent. of itself.

17.   If I give away 25 per cent. of my money to A, and 20 per cent. to B, and lend 55 per cent. to my brother, how much do I have left ?

18.   Bought a drove of cattle ; in the first town I passed through, I sold 25 per cent. of them, and afterwards lost 8 per cent. of them ; what per cent. of the drove had I then left ?

19.   If I draw 23 per cent. of my money out of the bank where it is placed, how much remains there ?

20.   If I borrow $70, and pay 6 per cent. a year for the use of it, how much do I pay in 6 years ?

21.   How much is 4 times 7 per cent. of $50 ?

22.   How much is 8 times 6 per cent. of $80 ?

23.   How much is 9 times 5 per cent. of $60 ?

24.   How much is $\frac{1}{2}$ per cent. of $100 ?   Of $10 ? Of $1 ?   Of 1 dime ?

NOTE.   1 per cent. may be found first, as in Ex. 1, and $\frac{1}{2}$ of 1 per cent. will be $\frac{1}{2}$ as much.

25.   How much is $\frac{3}{4}$ per cent. of $200 ?   Of $20 ? Of $12 ?   Of $1 ?

26.   How much is $5\frac{1}{8}$ times 3 per cent. of 150 pounds ?

27. How much is 6 times 5 per cent. of 225 yards? Of 150 yards?

28. What sum is allowed for selling goods to the amount of $500, at 2½ per cent. commission?

29. A city broker exchanged $400 on a country bank, at ⅛ per cent.; what did he get for his trouble?

30. A man having $200 in uncurrent bank-bills, paid 1½ per cent. to have them exchanged for current money; how much did he pay?

31. How much is 4¼ times 6 per cent. of 200? 10 times 5½ per cent. of 100?

LESSON XLI.

1. What fractional part of anything is 10 per cent. of it?

Ans. *10 per cent. is $\frac{10}{100}$, which, reduced to its lowest terms, is $\frac{1}{10}$.*

2. What fractional part of anything is 12½ per cent. of it?

Analysis. *$12\frac{1}{2} = \frac{25}{2}$; ½ of 1 per cent. is ½ of $\frac{1}{100}$, or $\frac{1}{200}$, and $\frac{25}{2}$ of 1 per cent. is 25 times $\frac{1}{200}$, or ⅛.*

3. What fractional part of a quantity is 20 per cent. of it? 25 per cent.? 50 per cent.? 5 per cent.? 75 per cent.? 40 per cent.? 80 per cent.? 15 per cent.? 60 per cent.?

4. What fractional part of a quantity is 33⅓ per cent. of it? 8⅓ per cent.? 37½ per cent.? 16⅔ per cent.? 66⅔ per cent.? 62½ per cent.? 87½ per cent.? 6¼ per cent?

5. What fraction is 4 times 6 per cent.?

6. What fraction is 5½ times 8 per cent.?

7.   What part of a ton is 24 per cent. of 75 per cent. of it ?

NOTE. $\frac{24}{100} = \frac{6}{25}$, and $\frac{6}{25}$ of 75 per cent. = 18 per cent.

8.   What part of a drove of sheep is 20 per cent. of 80 per cent. of it ?

9.   What part of a ship is 25 per cent. of 50 per cent. of it ?

10.   What per cent. of anything is $\frac{1}{4}$ of it ?

ANALYSIS. — *Since anything is* 100 *per cent. of itself,* $\frac{1}{4}$ *of anything must be* $\frac{1}{4}$ *of* 100 *per cent., or* 25 *per cent., of it.*

11.   How many per cent. of anything is $\frac{1}{5}$ of it ?

12.   How many per cent. of anything is $\frac{2}{5}$ of it ?

13.   A man lost $\frac{1}{2}$ of his money; what per cent. did he lose ?

14.   If a man pays $\frac{3}{5}$ of his yearly income for board, what per cent. does he have left for other purposes ?

15.   How many per cent. of a quantity is $\frac{5}{6}$ of it ? $\frac{3}{4}$ of it ? $\frac{4}{5}$ of it ? $\frac{3}{10}$ of it ? $\frac{2}{3}$ of it ? $\frac{2}{7}$ of it ? $\frac{5}{6}$ of it ? $\frac{7}{8}$ of it ? $\frac{9}{10}$ of it ? $\frac{6}{25}$ of it ? $\frac{1}{4}$ of $\frac{3}{5}$ of it ? $\frac{1}{3}$ of $\frac{1}{4}$ of it ?

16.   In a certain school, 20 per cent. are in the first class, $\frac{2}{5}$ of the remainder in the second class, and the rest in the third class, which has two equal divisions; what fractional part of the school is in each division ?

17.   What per cent. of 24 is 6 ?

ANALYSIS.   6 *is* $\frac{6}{24}$, *or* $\frac{1}{4}$, *of* 24; *and since* 24 *is* 100 *per cent. of itself,* 6 *must be* $\frac{1}{4}$ *of* 100 *per cent., or* 25 *per cent., of* 24.

The BASIS of percentage is the number on which percentage is reckoned. Thus, in Ex. 17, 24 is the basis of percentage.

18.   What per cent. of 10 is 2 ?   Of 50 is 5 ?

19. What per cent. of 25 is 5 ? 10 ? 8 ? 20 ?

20. What per cent. of 42 is 7 ? 14 ? 21 ? 35 ?

21. 7 is what per cent. of 14 ? Of 28 ? Of 49 ? Of 70 ? Of 140 ?

22. $ 25 is twice what per cent. of $150 ?

23. $\frac{1}{4}$ is what per cent. of $12\frac{1}{2}$?

Note. $12\frac{1}{2} = \frac{25}{2} = \frac{50}{4}$: and $\frac{1}{4}$ is $\frac{1}{50}$ of $\frac{50}{4}$.

24. $\frac{1}{5}$ is what per cent. of 4 ? Of 12 ? Of 20 ?

25. 1 is what per cent. of $2\frac{1}{2}$? Of $5\frac{1}{2}$? Of $6\frac{1}{4}$?

26. $\frac{2}{3}$ of $\frac{3}{4}$ is what per cent. of $\frac{1}{8}$ of 12 ?

27. $\frac{1}{2}$ of 16 per cent. is what per cent. of 24 per cent. ? Of 64 per cent. ?

28. $\frac{1}{3}$ of 15 per cent. is what per cent. of 18 per cent. ? Of 30 per cent. ?

29. $\frac{3}{4}$ of 16 per cent. is what per cent. of 24 per cent. ? Of 72 per cent. ?

30. If a miller takes out 4 quarts for every bushel he grinds, what per cent. toll does he take ?

31. If of a hogshead of sugar $\frac{1}{4}$ is sold, and of the remainder $\frac{1}{8}$ is rendered unsalable, what per cent. is the remainder ?

32. $\frac{4}{5}$ of $\frac{1}{2}$ is what per cent. of $\frac{2}{4}$?

33. Sold, from a box of sugar containing 150 pounds, at one time 20 pounds, and at another time 30 pounds ; what per cent. of the whole was sold ?

34. Of 120 yards of cloth there have been sold 108 yards ; what per cent. of the whole remains unsold ?

35. 25 per cent. of $33\frac{1}{3}$ per cent. is what per cent. of $16\frac{2}{3}$ per cent ?

36. $33\frac{1}{3}$ per cent. of 30 per cent. is what per cent. of $12\frac{1}{2}$ per cent. of 160 per cent. ?

37. What per cent. of anything is 50 per cent. of 25 per cent. of 80 per cent. of it ?

38. $\frac{2}{3}$ of 100 is what per cent. of $\frac{4}{5}$ of $88\frac{1}{5}$?

LESSON XLII.

1. 12 is 6 per cent. of what number?

ANALYSIS. — *Since* 6 *per cent. of some number is* 12, 1 *per cent. of that number is* ⅙ *of* 12, *or* 2, *and* 100 *per cent., or the number itself, is* 100 *times* 2, *or* 200.

2. 15 is 10 per cent. of what number?

ANALYSIS. $\frac{10}{100} = \frac{1}{10}$; *and if* $\frac{1}{10}$ *of some number is* 15, $\frac{10}{10}$, *or the number itself, is* 10 *times* 15, *or* 150.

3. 20 is 5 per cent. of what number? 4 per cent.? 10 per cent?

4. 1½ is 12 per cent. of what number? 3 per cent.? 15 per cent.?

5. ⅕ is 7 per cent. of what number? 6 per cent.? 4 per cent.?

6. 24 is 40 per cent. of what number? 12 per cent.? 25 per cent.?

7. Sold rye so as to gain 25 cents a bushel, which was 20 per cent. of what it cost; what did it cost?

8. Paid 80 cents for making a vest, which was 20 per cent. of the cost of the cloth; what was the cost of the cloth?

9. Bought a horse for $160, which was 20 per cent. less than his true value; what was his true value?

NOTE. — The expression "20 per cent. less than his true value" shows that his true value was the *basis* of the percentage. As his true value was 100 per cent. of itself, then $160 must be 100 per cent. — 20 per cent., or 80 per cent., of his true value.

10. If I buy a carriage for $228, at 24 per cent. less than what I can sell it for, what can I sell it for?

11. A sells a lot of dry goods for $60, at 20 per cent. above cost; what did the goods cost him?

NOTE. — As the cost was 100 per cent. of itself, $60 must be 120 per cent., or ⅘, of the cost. In buying and selling, the gain or loss per cent. is reckoned on the cost as a basis.

12. A merchant sold a lot of goods for $260, and thereby gained 30 per cent.; what was the cost?

13. Sold a coat for $7, and thereby lost 30 per cent.; what was the cost?

14. Sold 2 horses, at $200 apiece; on one there was a gain of 20 per cent., and on the other a loss of 20 per cent.; was there a gain, or a loss, on the sale of the two, and of how much?

15. An apple-woman bought apples at 60 cents a hundred, and sold them at 1 cent apiece; how much per cent. did she gain?

ANALYSIS. — *Since she bought apples at* 60 *cents a hundred, and sold them at* 100 *cents a hundred, she gained the difference between* 100 *cents and* 60, *cents, or* 40 *cents, on every* 60 *cents. As* 40 *cents are* $\frac{40}{60}$, *or* $\frac{2}{3}$, *of* 60 *cents, the gain is* $\frac{2}{3}$ *of* 100 *per cent., or* 66$\frac{2}{3}$ *per cent.*

16. When cloth is bought at $1.20 per yard, and sold at $1 per yard, how much per cent. is the loss?

17. Bought cloth at 80 cents per yard, and sold it at $1 per yard; how much per cent. was gained?

18. If a horse be bought for $80, and a cow for $25, and each be sold so as to gain 16 per cent., how much is received for them both?

NOTE. — Either find 16 per cent., or $\frac{4}{25}$, of the cost, and add it to the cost; or find 116 per cent., or $\frac{29}{25}$, of the cost.

19. John Robinson buys a cargo of coal for $525, and sells it so as to gain 12 per cent.; for how much does he sell it?

20. Having a house worth $1400, I charge for it 9\frac{1}{2}$ per month; how much per cent. a year do I get for it?

21. What per cent. does a merchant lose, who sells corn at 80 cents a bushel which cost him 90 cents?

22. Bought 12 barrels of flour for $110, and

sold the same again for $9.50 a barrel; how much was the gain or loss per cent.?

23. I have a house which brings in ⅖ of its value every 5 years; how much per cent. do I receive on it each year?

24. A bought goods for $100, and sold them to B so as to gain 10 per cent.; B sold them again to C, and gained 20 per cent.; how much did C· pay? How many ·dollars could A have made, if he had sold them himself to C, at the same price C paid?

25. A buys goods, and sells again· to B at 10 per cent. more than he gives; B sells to C, and makes 20 per cent.; how much ·per cent. would A make, were he to sell to C himself at the same price that B receives?

ANALYSIS. — *As A's cost is* 100 *per cent. of itself, B pays* 110 *per cent. of A's cost, and gains* 20 *per cent., or* ⅕, *of* 110 *per cent, which is* 22 *per cent., of A's cost; therefore A, by selling directly to C, would gain* 10 *per cent. and* 22 *per cent., or* 32 *per cent.*

26. A sells B a watch, and gains 10 per cent.; B sells it again to C, and gains 20 per cent.; C pays $110; how much did it cost A?

27. If A has 50 per cent. less capital than B, B has what per cent. more than A?

ANALYSIS. — *Since B's capital is* 100 *per cent of itself, and A's is* 50 *per cent. less than B's, A's must be* 50 *per cent. of B's and the difference between B's capital and A's must be* ½½ *of A's, or once A's. Therefore B has once* 100 *per cent., or* 100 *per cent., more capital than A.*

28. If A's capital is 50 per cent. of B's, B's is what per cent. of A's?

29. Charles Thompson sells to John Johnson goods which he bought for $100, and gains 10 per cent.; he also sells some of the same goods to

Peter Williams, and gains 30 per cent.; how many dollars would Johnson gain if he were to sell all he bought to Williams at the same price that Thompson sells to Williams?

30. If I sell to A, a retail dealer, and gain 10 per cent., and to B, who is one of A's customers, and gain 30 per cent., what per cent. would A gain by selling to B at the same price that I sell to B?

31. If James has 50 per cent. more money than John, what per cent. has John less than James? John's money is what per cent. of James's?

32. Having a farm of 154 acres, worth $20 an acre, I let half of it at 5 per cent. a year on its value, and cultivated the other half myself, getting back $115 beyond all expenses; is it better for me to let my farm, or use it myself, and how much difference is there?

33. Bought a horse for $200, which was 20 per cent. less than his worth, and sold him at 95 per cent. of his value; how many dollars were gained?

34. If a barrel of flour is bought for $\frac{5}{8}$ of its market price, and sold for 4 per cent. more than the market price, what per cent. is gained?

35. Bought a cargo of flour at 20 per cent. less than $7 a barrel, and sold it at 4 per cent. more than $7 a barrel; what per cent. was gained? How many dollars were gained on a barrel?

36. If Philadelphia had 25 per cent. less population than New York, New York would have how many per cent. more than Philadelphia?

37. Paid my agent, for the purchase of a horse, $168, including 5 per cent. allowed him for his services; what did he pay for the horse?

38. One publisher allows his agents 20 per cent. of all the money which they receive for his books, while another allows his agents to deduct

a sum equal to 25 per cent. of the amount which they remit to him; which terms are most favorable for the agents?

39. A grocer bought 100 eggs, at 15 cents a dozen, but 16 of them proved bad, and he sold the rest at 18 cents a dozen; how much per cent. did he gain?

40. Buying $150 worth of goods, I lose 14 per cent., but at the same time I buy $360 worth, and gain 9 per cent.; do I gain or lose on my whole capital, and how much?

41. A sells to B for $6 goods which he bought for $5; B sells again to C, and loses 10 per cent.; C sells to D at $7 what cost him $6; how much per cent. would A gain by selling directly to D.

42. A man gains 20 per cent., in each of three years, upon what he had at the beginning of the year; how much more has he then than when he began?

LESSON XLIII.

INTEREST is money paid for the use of money.

The PRINCIPAL is the sum of money for which interest is paid.

Interest is usually reckoned at a certain rate per cent. for one year. Thus, 6 per cent. means that $6 are paid for the use of $100 for 1 year, 6 cents for the use of 100 cents, or $1, and so on; or that the interest for 1 year is equal to $\frac{6}{100}$ of the principal.

1. What is the interest of $1 for 2 years at 6 per cent?

ANALYSIS. — *If the interest of $1 for 1 year is 6 cents, for 2 years it is 2 times 6 cents, or 12 cents.*

2. What is the interest of $1 for 3 years, at 6 per cent.? For 4 years? For 10 years?

3. What is the interest of $1 for 2 years, at 4 per cent.? For 5 years? For 8 years?

4. What is the interest of $10 for 1 year, at 5 per cent.?

FIRST ANALYSIS. — *If the interest of $1 for 1 year is 5 cents, the interest of $10 is 10 times 5 cents, or 50 cents.*

SECOND ANALYSIS. — *As the interest is equal to 5 per cent. of the principal, it is equal to $\frac{5}{100}$, or $\frac{1}{20}$, of the principal; $\frac{1}{20}$ of $10 is $\frac{1}{2}$, or 50 cents.*

5. What is the interest of $2 for 1 year, at 6 per cent.? At 5 per cent.? At 8 per cent.?

6. What is the interest of $4 for 1 year, at 6 per cent.? Of $5? Of $7? Of $10?

7. What is the interest of $75 for 1 year, at 8 per cent.? At 10 per cent.? At 12 per cent.?

NOTE. — Apply the second analysis to these examples.

8. What is the interest of $125 for 1 year, at 6 per cent.? At 5 per cent.? At 7 per cent.?

9. What is the interest of $50 for 1 year, at 6 per cent.? At 4 per cent.? At 8 per cent.?

10. What is the interest of $5 for 4 years, at 6 per cent.?

FIRST ANALYSIS. — *If the interest of $1 for 1 year is 6 cents, for 4 years it is 4 times 6 cents, or 24 cents; and if the interest of $1 for 4 years is 24 cents, the interest of $5 for 4 years is 5 times 24 cents, which are 120 cents, or $1.20.*

SECOND ANALYSIS. — *As the interest for 1 year is equal to 6 per cent. of the principal, for 4 years it is equal to 4 times 6 per cent., which are 24 per cent, or $\frac{24}{100}$, of the principal; $\frac{24}{100}$ of $5 is $\frac{1}{5}$, and $\frac{24}{100}$ of $5 are $\frac{6}{5}$, or $1\frac{1}{5}$.*

NOTE. — The second method will generally be found preferable for mental operations, unless the principal is quite small.

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11. What is the interest of $4 for 2 years, at 6 per cent.? Of $6? Of $8?

12. What is the interest of $10 for 5 years, at 4 per cent.? At 3 per cent.? At 5 per cent.?

13. What is the interest of $25 for 6 years, at 6 per cent.? At 8 per cent.? At 10 per cent.?

14. What is the interest of $10 for 3 years, at 6 per cent.? Of $12? Of $20?

15. What is the interest of $50 for 5 years, at 6 per cent.? For 4 years?

16. What is the interest of $600 for 5 years, at $3\frac{1}{8}$ per cent.? At $6\frac{2}{3}$ per cent.? At 6 per cent.?

17. What is the interest of $400 for $7\frac{1}{2}$ years, at 5 per cent.? At 8 per cent.?

18. What is the interest of $60 for 7 years, at 6 per cent.? For 9 years?

19. What is the interest of $240 for 10 years, at 5 per cent.? At $7\frac{1}{2}$ per cent.? At 4 per cent.?

20. What is the interest of $3 for 3 years, at 8 per cent.? Of $3\frac{1}{3}$? Of $5\frac{1}{4}$? Of $6\frac{1}{4}$?

21. What is the interest of $15 for 8 years, at 10 per cent.? At 4 per cent.? At 6 per cent.?

22. What is the interest of $100 for 5 years, at 7 per cent.? For 6 years? For 7 years?

23. What is the interest of $150 for 2 years, at 7 per cent.? For 3 years? For 4 years?

24. What is the interest of $200 for 3 years, at 7 per cent.? At 8 per cent.?

25. What is the interest of $320 for 4 years, at 10 per cent.? At $12\frac{1}{2}$ per cent.?

26. What is the interest of $220 for 5 years, at 7 per cent.? Of $500? Of $600?

27. What is the interest of $100 for 1 month, at 6 per cent.?

FIRST ANALYSIS. — *If the interest of* $100 *for* 1 *year is* $6, *for* 1 *month, or* $\frac{1}{12}$ *of a year, it is* $\frac{1}{12}$ *of* $6, *which is* $\frac{1}{2}$, *or* 50 *cents.*

SECOND ANALYSIS. — *As the interest for 1 year is equal to 6 per cent. of the principal, for 1 month, or $\frac{1}{12}$ of a year, it is equal to $\frac{1}{12}$ of 6 per cent., which is $\frac{1}{2}$ of 1 per cent., or $\frac{1}{200}$ of the principal; $\frac{1}{200}$ of $ 100 is $$\frac{1}{2}$, or 50 cents.*

28. What is the interest of $1 for 1 month, at 6 per cent. ? Of $10 ? Of $6 ? Of $8 ?

29. What is the interest of $1 for 1 month, at 12 per cent. ? At 9 per cent. ? At 8 per cent. ? At 7 per cent. ? At 4 per cent. ? At 5 per cent. ? At 10 per cent. ? At 3 per cent.?

30. What is the interest of $60 for 2 months, at 6 per cent. ? Of $9 ? Of $90 ?

NOTE. 2 months = $\frac{1}{6}$ of a year.

31. What is the interest of $200 for 1 month, at 6 per cent. ? Of $300 ? Of $360 ? Of $420 ?

32. What is the interest of $100 for 4 months, at 6 per cent. ? For 5 months? For 6 months? For 8 months ? For 10 months ?

33. What is the interest of $100 for 6 months, at 7 per cent. ? At 8 per cent. ? At 12 per cent. ?

34. What is the interest of $100 for 4 years and 4 months, at 6 per cent. ? Of $200 ? Of $300 ? Of $50 ? Of $850 ?

NOTE. 4 years 4 months = $4\frac{1}{3}$ years, or $\frac{13}{3}$ of a year.

35. What is the interest of $50 for 4 years and 3 months, at 4 per cent. ? At 8 per cent. ?

36. What is the interest of $150 for 6 years and 8 months, at 6 per cent.? For 4 years 2 months?

37. What is the interest of $468 for 8 years and 4 months, at 5 per cent.? For 1 year 8 months?

38. What is the interest of $625 for 5 years and 6 months, at 8 per cent.?

39. What is the interest of $800 for 1 year and 7 months, at 3 per cent. ?

LESSON XLIV.

In reckoning interest, a month is usually considered as consisting of 30 days, and a year of 360 days.

1. What is the interest of $1 for 30 days, at 6 per cent.? For 60 days? For 120 days? For 20 days? For 40 days? For 90 days? For 12 days? For 15 days? For 100 days?

NOTE. 30 days $= \frac{30}{360}$, or $\frac{1}{12}$, of a year; 60 days $= \frac{60}{360}$, or $\frac{1}{6}$, of a year, and so on.

2. What is the interest of $60 for 24 days, at 7 per cent.? For 45 days? For 18 days? For 36 days? For 72 days?

3. What is the interest of $150 for 1 month and 24 days, at 8 per cent.?

ANALYSIS. — *As there are 30 days in 1 month, 1m. 24d.* $= 1\frac{24}{30}m. = 1\frac{4}{5}m. = \frac{9}{5}m.$, *and as* $1m. = \frac{1}{12}y.$, $\frac{9}{5}m. = \frac{9}{5}$ *of* $\frac{1}{12}y. = \frac{9}{60}y. = \frac{3}{20}y.$
As the interest for 1 year is equal to 8 per cent. of the principal, for $\frac{3}{20}$ *of a year, it is equal to* $\frac{3}{20}$ *of 8 per cent.;* ($\frac{1}{20}$ *of 8 per cent. is* $\frac{8}{20}$, *or* $\frac{2}{5}$, *per cent., and* $\frac{3}{20}$ *are 3 times* $\frac{2}{5}$ *per cent.*), *or* $\frac{6}{5}$ *per cent., that is,* $\frac{6}{500}$, *or* $\frac{3}{250}$ *of the principal.* $\frac{1}{250}$ *of* 150 *is* $\frac{1}{2}\frac{2}{3}$, *or* $\frac{3}{5}$, *and* $\frac{3}{250}$ *are 3 times* $\frac{3}{5}$, *which are* $\frac{9}{5}$, *or* $1.80.$

4. What is the interest of $300 for 7 months and 10 days, at 6 per cent.?

5. What is the interest of $175 for 9 months and 18 days, at 5 per cent.?

6. What is the interest of $50 for 10 months and 20 days, at 6 per cent.?

7. What is the interest of $100 for 4 months and 15 days, at 7 per cent.?

8. What is the interest of $125 for 4 years, 2 months, and 12 days, at 5 per cent.?

NOTE. 4y. 2m. 12d. $= 4\frac{1}{5}y. = \frac{21}{5}y.$

9. What is the interest of $ 300 for 3 years, 3 months, and 10 days, at 6 per cent.?

10. What is the interest of $ 150 for 1 year and 20 days, at 6 per cent.?

11. What is the interest 'of $ 30 for 6 years, at 5 per cent.? At 7 per cent.?

12. What is the interest of $ 10 for 4 years and 3 months, at 4 per cent.? At 8 per cent.?

13. What is the interest of $ 150 for 5 years and 4 months, at 6 per cent.?

14. What is the interest of $ 360 for 1 year, 4 months, and 20 days, at 6 per cent.?

15. What is the interest of $ 45 for 4 years, 5 months, and 10 days, at 6 per cent.?

16. What is the interest of $ 80 for 5 years, 7 months, and 15 days, at 6 per cent.?

17. What is the interest of $ 600 for. 2 years, 1 month, and 18 days, at 5 per cent.?

18. What is the interest of $ 800 for 9 months and 10 days, at 9 per cent.?

19. What is the interest of $ 210 for 3 years and 4 months, at 7 per cent.?

20. What is the interest of $ 10 for 1 day, at 6 per cent.? For 6 days ? For 10 days ?

ANALYSIS. — *If the interest of $ 1 for 1 year is 6 cents, for 1 month it is $\frac{1}{12}$ of 6 cents, which is $\frac{1}{2}$ of a cent, or 5 mills, and for 1 day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill; and if the interest of $ 1 for 1 day, at 6 per cent., is $\frac{1}{6}$ of a mill, the interest of $ 10 is 10 times $\frac{1}{6}$ of a mill, which are 1$\frac{2}{3}$ mills, or $\frac{1}{6}$ of a cent.*

NOTE. — This form of analysis may be used when the days, or months and days, cannot be reduced to a convenient fraction of a year.

As 6 days and 10 days are, respectively, $\frac{1}{5}$ and $\frac{1}{3}$ of a month, these fractions may be used in place of $\frac{1}{30}$, in finding the interest for 6 days and 10 days; or, the interest for 6 days and 10 days may be obtained from the interest for 1 day, after that is known.

21. What is the interest of $ 20 for 5 days, at 6 per cent. ? For 1 day ? For 3 days ?

22. What is the interest of $ 1 for 2 days, at 12 per cent. ? For 1 day ? For 14 days ?

23. What is the interest of $ 5 for 4 days, at 6 per cent. ? For 4 months ? For 4 months and 4 days ?

24. The interest of $ 240 for a certain time is $ 18, at 6 per cent. ; how much is it at 8 per cent. ?

NOTE. — Either add to the interest at 6 per cent. $\frac{1}{3}$ of itself, or find from it the interest at 1 per cent. and then at 8.

25. Find the interest of $ 40 for 4 years and 2 months, at 6 per cent. At 4 per cent. At 5 per cent. At 7 per cent. At 8 per cent.

26. Find the interest of $ 10 for 8 months and 12 days, at 6 per cent. At 4 per cent. At 5 per cent. At 7 per cent. At 8 per cent.

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LESSON XLV.

The AMOUNT is the interest and principal added together.

1. What is the amount of $ 40 for 2 years, at 6 per cent. ?

FIRST SOLUTION. — *The interest, which may be obtained by either of the methods mentioned in the previous lessons, is $ 4.80, and $ 40 + $ 4.80 = $ 44.80, the amount.*

SECOND SOLUTION. — *The interest for 2 years, at 6 per cent., is equal to $\frac{12}{100}$, or $\frac{3}{25}$, of the principal; and, as the principal is $\frac{25}{25}$ of itself, the amount is equal to $\frac{25}{25} + \frac{3}{25}$, or $\frac{28}{25}$, of the principal. $\frac{1}{25}$ of $ 40 is $ 1\frac{3}{5}$, or $ \frac{8}{5}$, and $\frac{28}{25}$, or the amount, is 28 times $ \frac{8}{5}$, which are $ 2\frac{34}{4}$, or $ 44\frac{4}{5}$.*

2. What is the amount of $ 60 for 4 years, at 4 per cent. ?

3. What is the amount of $ 80 for 3 years, at 5 per cent. ? For 4 years ?

4. What is the amount of $100 for 6 years, at 4 per cent. ? At 6 per cent.?

5. What is the interest of $70 for 6 years, at 7 per cent. ? At 5 per cent.?

6. What is the amount of $150 for 2 years, at 7 per cent. ? For 4 years?

7. What is the interest of $100 for 3 years and 4 months, at 7 per cent. ?

8. What is the interest of $25 for 6 years and 1 month, at 6 per cent. ?

9. What is the interest of $10 for 4 years and 2 months, at 12 per cent. ?

10. What is the amount of $150 for 4 years and 1 month, at 6 per cent. ?

11. Required the amount of $ 200 for 2 years and 2 months, at 4 per cent.

12. Required the amount of $145 for 6 years and 3 months, at 8 per cent.

13. Required the amount of $400 for 2 years, 2 months, and 20 days, at $4\frac{1}{2}$ per cent.

14. What is the amount of $140 for 2 years, 4 months, and 24 days, at 5 per cent. ?

15. What is the amount of $ 200 for 2 years, 6 months, and 15 days, at 6 per cent. ?

16. What is the interest of $15 for 6 years, 10 months, and 15 days, at 8 per cent. ?

17. What is the amount of $ 360 for 2 years and 1 month, at 6 per cent. ?

18. What is the amount of $180 for 6 years and 8 months, at 10 per cent. ?

19. What is the amount of $ 200 for 5 years and 9 months, at 4 per cent. ?

20. What is the amount of $140 for 8 years, 1 month, and 15 days, at 8 per cent. ?

21. What is the amount of $175 for $\frac{4}{7}$ years and 8 months, at 6 per cent. ?

22. What is the amount of $140 for 3 years, and 4 months, at 3 per cent. ?

23. Required the amount of $160 for 9 years, at 5 per cent.

24. Required the amount of $300 for 5 years, 6 months, and 20 days, at 6 per cent.

25. What is the amount of $250 for 1 year, 6 months, and 24 days, at 6 per cent. ?

26. What is the amount of $500 for 1 year, 3 months, and 18 days, at 10 per cent. ?

LESSON XLVI.

1. If the interest of $120 for 1 year is $9, what is the interest of $60 ?

ANALYSIS BY RATIO.* $60 *is* $\frac{1}{2}$ *of* $120, *and if the interest of* $120 *is* $9, *the interest of* $60 *is* $\frac{1}{2}$ *of* $9, *or* $4\frac{1}{2}$.

ANALYSIS BY UNITY. — *If the interest of* $120 *is* $9, *the interest of* $1 *is* $\frac{1}{120}$ *of* $9, *or* $\frac{3}{40}$, *and the interest of* $60 *is* 60 *times* $\frac{3}{40}$, *which are* $\frac{180}{40}$, *or* $4\frac{1}{2}$.

2. If the interest of $400 for 1 year is $24, what is the interest of $100 ?   Of $10 ?   Of $1 ?

3. If the interest of $12 for 5 years is $3.60, what is the interest of $6 ?   Of $60 ?   Of $300 ?

4. If the interest of $160 for 2 years is $18, what is the interest of $80 ?   Of $40 ?

5. If the interest of $280 for 3 years is $46, what is the interest of $70 ?   Of $140 ?

6. If the interest of $100 for 3 years is $20,

* See Lesson XXXVII.

what will it be for 6 years? For 1 year? For 10 years? Fór 4 years?

7. If the interest of $180 for 2 years is $23, what is it for 3 years? For 7 years? For 9 years?

8. If the interest of $200 for 3 years is $27, what is it for 1 year? For 1 month? For 4 months? For 7 months?

9. If the interest of $200 for 1 month is $1, what is it for 15 days? For 5 days? For 1 day?

10. If the interest of $140 for 1 year and 4 months is $10, what is it for 8 months? For 4 months? For 2 months? For 1 month?

11. The interest of $240, at 6 per cent., is $27; what is it at 2 per cent.? At 1 per cent.?

12. The interest of $320, at 7 per cent., is $15; what is it at 1 per cent.? At 8 per cent.?

13. If the interest of $100 for 3 years is $21, what is the rate per cent.?

14. If the interest of $300 for 1 year is $21, what is the rate per cent.?

15. What is the interest of $100 for 4 years, at 1 per cent.? If the interest of $100 for 4 years is $20, what is the rate per cent.?

16. If the interest of $300 for 2 years is $48, what is the rate per cent.?

FIRST ANALYSIS. — *If the interest of $300 for 2 years is $48, for 1 year it is ½ of $48, or $24; and as $24 are equal to $\frac{24}{300}$, or $\frac{8}{100}$, of the principal, the rate per cent. must be* 8.

SECOND ANALYSIS. — *The interest of $300 for 2 years, at 1 per cent., is $6, and since $6 is the interest at 1 per cent., $48 must be the interest at as many per cent. as $6 are contained times in $48, which are* 8.

17. If the interest of $200 for 2 years is $12, what is the rate per cent.?

6 *

18. If the interest of $ 400 for 3 years is $ 24, what is the rate per cent. ?

19. If the interest of $ 25 for 1 year and 8 months is $ 1, what is the rate per cent. ?

FIRST ANALYSIS. $1y. \ 8m. = 1\frac{2}{3}y. = \frac{5}{3}y.$ *If the interest of $ 25 for $\frac{1}{3}$ of a year is $ 1, for $\frac{1}{3}$ of a year it is $ $\frac{1}{3}$, and for $\frac{2}{3}$, or a year, it is $ $\frac{2}{3}$. As the principal is $ 25, or $ $\frac{125}{5}$, the interest for 1 year is equal to $\frac{3}{125}$ of the principal, and since the principal is 100 per cent. of itself, the interest for 1 year is $\frac{3}{125}$ of 100 per cent., which are $\frac{12}{5}$ per cent., or $2\frac{2}{5}$ per cent.*

SECOND ANALYSIS. — *The interest of $ 25 for 1 year and 8 months, at 1 per cent., is $ $\frac{5}{12}$; and since $ $\frac{5}{12}$ is the interest at 1 per cent., $1 must be the interest at as many per cent. as $ $\frac{5}{12}$ are contained times in $ 1, or $ $1\frac{2}{5}$, which are $2\frac{2}{5}$. Therefore $2\frac{2}{5}$ per cent. is the required rate.*

20. If the interest of $150 for 3 years and 4 months is $ 35, what is the rate per cent.?

21. If the interest of $ 50 for 1 year and 3 months is $ 3, what is the rate per cent.?

22. If the interest of $ 75 for 4 years and 2 months is $ 12½, what is the rate per cent.?

23. A man paid $ 8 for the use of $ 48 for 1 year and 4 months ; what was the rate per cent.?

24. John Niles lends Harry Hubbard $ 30 for 2 years and 6 months ; Harry has to pay at the end of the time $ 36 ; what is the rate per cent.?

NOTE. — The principal, $ 30, has to be paid back, and is of course a part of the $ 36 ; the remainder, $ 6, is the interest, by which the rate per cent. is to be found.

25. A note of $ 100, being on interest 2 years and 2 months, amounted to $ 126 ; what was the rate per cent.?

. 26. A gentleman lent $ 60 for 1 year and 6 months, and received $ 90 ; what was the rate per cent.?

LESSON XLVII.

. 1. What principal, in 4 years, at 10 per cent., will give $ 6 interest ?

FIRST ANALYSIS. — *The interest for 4 years, at 10 per cent., is equal to $\frac{40}{100}$, or $\frac{2}{5}$, of the principal; then if $\frac{2}{5}$ of the principal are $ 6, $\frac{1}{5}$ is $\frac{1}{2}$ of $ 6, or $ 3, and $\frac{5}{5}$, or the principal itself, is 5 times $ 3, or $ 15.*

SECOND ANALYSIS. — *The interest of $ 1, for 4 years, at 10 per cent., is $ $\frac{2}{5}$, and since $ $\frac{2}{5}$, or 4 dimes, is the interest of $ 1, $ 6, or 60 dimes, must be the interest of as many dollars as 4 dimes are contained times in 60 dimes, which are 15.   Therefore the required principal is $ 15.*

2. What principal, in 2 years, at 4 per cent., will give $ 8 interest ?

3. What is the principal that, in 4 years, at 3 per cent., will give $ 6 interest ?

4. What principal, in 5 years, at 6 per cent., will give $ 10 interest ?

5. What principal is sufficient in 3 years, 2 months, and 12 days, at 5 per cent., to gain $ 64 ?

6. What principal is sufficient in 2 years, at 8 per cent., to gain $ 10 ?

7. What principal is sufficient in 6 years, at 1 per cent., to gain $ 20 ?

8. What principal, in 1 year and 8 months, at 4 per cent., will gain $ 5 ?

9. What principal, in 6 years and 4 months, at 6 per cent., will gain $ 19 ?

10. What principal is sufficient in 7 years, to gain $ 14, at 7 per cent.?

11. The interest on a note for 4 years and 2 months, at 4 per cent., was $ 60 ; what was the principal ?

12. If the interest of $50, at 6 per cent., is $6, how long has it been on interest?

ANALYSIS. — *The interest of $50, for 1 year, at 6 per cent., is $3; and since $3 is the interest for 1 year, $6 must be the interest for as many years as $3 are contained times in $6, which are 2. Therefore 2 years is the time required.*

13. If the interest of $60, at 2 per cent., is $12, how long has it been on interest?

14. How long must $100 be on interest, at 4 per cent., to gain $40?

15. A note of $80, being on interest at 8 per cent., amounted to $160; how long was it on interest?

16. How long must $10 be on interest, at 5 per cent., to gain $3?

17. If the interest of $20, at 4 per cent., is $4, how long a time has it been on interest?

18. If the interest of $50, at 8 per cent., is $12, how long has it been on interest?

19. Required the time that $40 must be on interest, at 2 per cent., to gain $8.

20. A gentleman lent $60, at 6 per cent., and received $140; how long was it on interest?

21. A sum of money is on interest at 6 per cent.; how long will it take it to double itself?

NOTE. — That is, if it gain 6 per cent. a year, how long will it take to gain 100 per cent.?

22. How long will it take a sum of money to double itself, at 9 per cent.?   At 18 per cent.?

23. A given principal gains $\frac{1}{4}$ of $\frac{3}{7}$ of itself a year; how long will it take it to double itself?

NOTE. — That is, how long will it take to gain $\frac{24}{24}$ of itself?

24. A given principal gains $\frac{2}{5}$ of $\frac{1}{3}$ of itself a year; how long will it take to double itself?   To gain $\frac{1}{2}$ of itself? $\frac{1}{4}$ of itself?

25. If I loan $1200, at 8 per cent. a year, how long will it be in gaining $80?

26. If I loan $500, at 7 per cent. a year, how long will it be in gaining $350?

LESSON XLVIII.

The PRESENT WORTH of a sum due at some future time is what should be paid now, instead of paying that sum then; it is therefore equivalent to a principal which, being put at interest, will amount to the debt at the time of its becoming due.

DISCOUNT is the allowance or deduction made for paying money before it is due; it is, therefore, equivalent to the interest on the present worth of the debt up to the time of its becoming due.

1. What principal, in 4 years, at 5 per cent., will amount to $96?

FIRST ANALYSIS. — *The interest for 4 years, at 5 per cent., is equal to $\frac{20}{100}$, or $\frac{1}{5}$ of the principal, and as the principal is $\frac{5}{5}$ of itself, the amount must be equal to $\frac{6}{5}$ of the principal; then, if $\frac{6}{5}$ of the principal are $96, $\frac{1}{5}$ of the principal is $\frac{1}{5}$ of $96, or $16, and $\frac{5}{5}$, or the principal itself, is 5 times $16, or $80.*

SECOND ANALYSIS. — *The amount of $1 for 4 years, at 5 per cent., is $\frac{6}{5}$, and since $\frac{6}{5}$, or 12 dimes, is the amount of $1, $96, or 960 dimes, must be the amount of as many dollars as 12 dimes are contained times in 960 dimes, which are 80. Therefore the required principal is $80.*

2. What principal, in 5 years and 10 months, at 6 per cent., will amount to $9?

3. What is the present worth of $50, due in 5 years, at 5 per cent.?

NOTE. — That is, what principal, put at interest for 5 years, at 5 per cent., will amount to $50?

4. What is the present worth of $136, due in 6 years, at 6 per cent.?

5. What is the present worth of $172, due in 9 years, at 8 per cent.?

6. What is the present worth of $7½, due in 5 years, at 10 per cent.?

7. What is the present worth of $78, due in 1 year, at 4 per cent.?

8. What is the present worth of $85, due in 10 years, at 7 per cent.?

9. What is the present worth of $96, due in 8 years, at 7½ per cent.?

10. What is the discount of $84, due in 8 years, at 5 per cent.?

FIRST SOLUTION. — *The present worth, which may be obtained by either of the methods given in this lesson, is* $60, *and* $84 — $60 = $24, *the discount.*

SECOND SOLUTION. — *The discount for 8 years, at 5 per cent., is equal to* $\frac{40}{100}$, *or* $\frac{2}{5}$, *of the present worth, and as the present worth is* $\frac{5}{5}$ *of itself, the given sum must be equal to* $\frac{7}{5}$ *of the present worth ; then, if* $\frac{7}{5}$ *of the present worth are* $84, $\frac{1}{5}$ *of the present worth is* $\frac{1}{7}$ *of* $84, *or* $12, *and* $\frac{2}{5}$ *of the present worth, or the discount, is* 2 *times* $12, *or* $24.

11. What is the discount of $100, due in 5 years, at 5 per cent.?

12. What is the discount of $77, due in 9 years, at 6 per cent.?

13. What is the discount of $66, due in 6½ years, at 10 per cent.?

14. What is the discount of $74, due in 6 years, at 8 per cent.?

15. What is the discount of $81, due in 5 years, at 7 per cent.?

16. What is the discount of $77⅛, due in 4 years, at 10 per cent.?

17. What is the discount of $57, due in 2 years, at 7 per cent.?

18. What is the discount of $63, due in 10 months, at 6 per cent.?

19. What is the discount of $81, due in 1 year and 4 months, at 6 per cent.?

20. What is the present worth of $64 due in 5 years, 7 months, and 6 days, at 5 per cent.?

21. What is the present worth of $122, due in 100 days, at 6 per cent.?

22. What are the present worth and discount of $108, due in 2 years and 11 months, at 12 per cent.?

23. What are the present worth and discount of $200, due in 6 years, 8 months, at 5 per cent.?

24. What are the present worth and discount of $50, due in 11 years, 1 month, and 10 days, at 6 per cent.?

LESSON XLIX.

1. Divide a sum of money between A and B, giving A $3 as often as you give B $2; what share of the money will each receive?

ANALYSIS. — *As often as A receives $3 and B $2, they both receive* $3 + $2, *or* $5; *therefore A receives ⅗, and B ⅖, of the money.*

2. If you divide $24 between A and B, by giving A $3 as often as you give B $5, how many dollars will each receive?

3. Divide $35 between John Wilson and Simon Edwards; how much will each get, if John receives $4 as often as Simon receives $3?

4. A man has two sons, to whom he leaves $660, to the elder $7 to every $4 he leaves the *younger;* how much does each get?

5. Divide 51 into two numbers which shall be to each other as $\frac{2}{3}$ to $\frac{3}{4}$.

ANALYSIS. $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$; *hence* $\frac{2}{3}$ *and* $\frac{3}{4}$ *are to each other as* $\frac{8}{12}$ *to* $\frac{9}{12}$, *or as* 8 *to* 9. *We therefore divide* 51 *into* 8 + 9, *or* 17, *equal parts, and one number will be* $\frac{8}{17}$ *and the other* $\frac{9}{17}$, *of* 51; $\frac{1}{17}$ *of* 51 *is* 3, $\frac{8}{17}$ *are* 8 *times* 3, *or* 24, *the first number, and* $\frac{9}{17}$ *are* 9 *times* 3, *or* 27, *the second number.*

6. Divide 60 into two numbers which shall be to each other as 7 to 5. As 3 to $\frac{1}{8}$. As $5\frac{1}{2}$ to 2.

7. Divide 69 into two parts which shall be to each other as $\frac{5}{6}$ to $\frac{4}{5}$.

8. Two men buy a cask of beer, containing 30 gallons; one pays $\$4$, and the other $3\frac{1}{2}$; what part does each get?

9. I go into partnership with Thomas Gould, and pay in $\$5$ of capital to every $\$4$ he pays; how much more do I pay than he? How much less does he pay than I? What share of the whole do I pay, and what share does he pay? If we have a profit of $\$63$, how much should I receive? How much should Gould receive?

10. Three men hired a pasture for $\$36$; A put in 4 oxen, B 3 oxen, and C 5 oxen; how much should each pay?

11. John Stevens and Samuel Judkins hire a pasture together; John puts in 3 cows for 2 days, and Samuel 4 cows for 6 days; they pay $\$5$ rent; how much should each pay?

NOTE.   3 cows for 2 days would be the same as 2 times 3 cows for 1 day.

12. Divide 55 into two parts, of which the larger is $\frac{6}{5}$ of the less; how great is each?

NOTE. — As the less is $\frac{5}{5}$ of itself, 55 must be $\frac{11}{5}$ of the less.

13. Divide 27 into two parts, of which one is $\frac{4}{5}$ of the other.

14. A gentleman has 3 sons, to whom he gives 98 cents to celebrate the Fourth of July; Edward, the eldest, has 2 times as much as Robert, the second, and 4 times as much as John, the youngest: how much does each receive?

NOTE. — John's share is once itself, Edward's is 4 times John's share, and Robert's is ½ of Edward's, or 2 times John's; therefore the whole sum, or 98 cents, is 7 times John's share.

15. Mary had 40 apples; she gave $\frac{2}{5}$ to her schoolmates, and divided the rest between her two sisters and herself, taking only $\frac{1}{5}$ as many as both her sisters; how many did she have for herself?

16. In a granary there is twice as much rye as wheat, twice as much wheat as buckwheat, and $\frac{1}{3}$ as much barley as rye; there are 125 bushels in all; how much of each kind?

17. While traveling, I met 11 beggars; 2 were cripples, 3 were blind, and 6 were too lazy to work; I gave each cripple 2 times as much as each blind man, and each blind man 3 times as much as each lazy man; in all I gave away 54 cents; how much did each cripple receive?

18. Two men entered into partnership; the first put in $500 for $4\frac{1}{2}$ months, and the second $600 for $3\frac{1}{3}$ months; their profits are $110; how much ought each to have?

NOTE. — Their shares of the profits must be to each other as 2400 to 2000, or 6 to 5. (See Example 11.)

19. Two men go into partnership; A puts in $200, and B $150; their profits are $32; how much per cent. do they gain on their capital, and what is each one's share?

20. A, B, and C enter into partnership; A puts in $\frac{1}{4}$, B $\frac{2}{5}$, and C the remainder; after a while A withdraws his, and the capital is now $480; how much does each put in?

21. C and D hire a pasture together; C pays $12, and D $10; C puts in 6 cows; how many should D put in?

22. What two fractions, whose sum is $1\frac{2}{5}$, are to each other as $2\frac{1}{2}$ to $3\frac{1}{2}$?

23. Two men start from New York, and travel in opposite directions, one at the rate of $4\frac{1}{4}$ miles an hour, and the other at the rate of $6\frac{3}{4}$ miles an hour; how far apart will they be at the end of 1 hour? How far in 4 hours? 6 hours? 10 hours?

24. A gentleman wished to give $100 for benevolent objects; having given away $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of it, he divided the rest equally among 5 poor widows; how much did each widow receive?

25. A man having returned from California, being asked how much money he had made, answered, that if he had made as much more, and half as much more, he should have $1000; how much had he made?

LESSON L.

1. A cistern is filled by a pipe in $4\frac{1}{2}$ hours; what part of it is filled in 1 hour?

2. A man can do $\frac{1}{5}$ of a piece of work in 1 hour; how long will it take him to do the whole?

3. Two men together can eat $\frac{1}{20}$ of a barrel of crackers in 1 day; how long will it take them to eat the whole?

4. A can do a piece of work in 8 days, and B the same work in 12 days; how long will it take them both?

ANALYSIS. — *Since A can do $\frac{1}{8}$ of the work in 1 day, and B $\frac{1}{12}$, both can do $\frac{1}{8} + \frac{1}{12}$, or $\frac{3}{24} + \frac{2}{24} = \frac{5}{24}$, in 1*

day ; and it will take them both as many days to do the whole work as $\frac{1}{24}$ is contained times in $\frac{3}{4}$, which are $\frac{24}{4}$, or $4\frac{4}{5}$. Therefore it would take them both $4\frac{4}{5}$ days to do the work.

NOTE. — The last part of the analysis may be thus expressed : since they can do $\frac{1}{24}$ of the work in 1 day, they can do $\frac{1}{24}$ of it in $\frac{1}{4}$ of a day, and $\frac{24}{24}$, or the whole work, in 24 times $\frac{1}{4}$ of a day, which are $\frac{24}{4}$ of a day, or $4\frac{4}{5}$ days.

5. Two men can each do $\frac{2}{7}$ of a piece of work in 1 day; how long will it take them both to do the whole ?

6. Two men set out to mow a field; the first can mow it in 16 days, and the second in 20 ; how long will it take them both to mow the field ?

7. A cistern has 3 pipes ; the first will fill it in 2 hours, the second in 3 hours, and the third in 6 hours ; how long will it take them all to fill it ?

8. A cistern has 2 pipes ; the first will fill it in 2 hours, and the second will empty it in 3 hours ; if both pipes are open, how long will it take to fill the cistern ?

9. A cistern has 4 pipes ; the first will fill it in 2 hours, the second will fill it 3 hours, the third in 4 hours, and the fourth will empty it in 2 hours ; now, if the pipes are all open at the same time, how long will it take to fill the cistern ?

10. A and B together can build a wall in 16 days, but with the aid of C they can build it in 10 days ; how long will it take C to build it alone ?

11. A can reap a certain piece of rye in $\frac{2}{5}$ of a day, B in $\frac{3}{4}$ of a day, and C in 1 day ; how long will it take them together to finish the piece, after C has been reaping $\frac{1}{2}$ of a day ?

12. A can cut a cord of wood in $\frac{2}{3}$ of a day, B in $\frac{3}{4}$ of a day, and C in $\frac{3}{5}$ of a day ; after A and B have cut $\frac{1}{2}$ of a day, how long will it take C to finish the *cutting* of the remainder of the cord ?

13. Divide 24 into 2 parts which shall be to each other as $1\frac{3}{4}$ to 3.

14. Divide 32 into 3 parts which shall be, respectively, as $2\frac{1}{3}$, $1\frac{3}{4}$, and $1\frac{1}{4}$.

15. A and B can do a piece of work in 15 days, and B alone in 24 days; how long would it take A alone?

16. Three men go into partnership; the first pays $7 as often as the second pays $4, and the third $5 as often as the second pays $8; they all pay in $540; how much does each pay?

17. Divide 86 into two parts, of which the larger is $5\frac{1}{2}$ times as much as the less.

18. Divide 50 into three parts, of which the first is $\frac{2}{3}$ as large as the second, and the third 3 times as large as the first and second.

19. Three men, A, B, and C, were to share $400 in the proportion of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, respectively; but, as C died, it is required to divide the whole sum properly between the other two; how much should each receive?

20. Two men, A and B, invest the same amount of money in a joint speculation, but A's money is used 3 months and 10 days, while B's is used 2 months and 15 days; if they gain $140, how much should each receive?

21. A man, in distributing some money to several indigent persons, gave $$2\frac{1}{2}$ to one man, $$3\frac{1}{3}$ to another, $$4\frac{3}{4}$ to another, $$5\frac{1}{6}$ to another, and $$1\frac{3}{8}$ to another; how many dollars did he give away?

22. Bought 12 buffalo robes, at $$12\frac{1}{2}$ apiece, and paid for them with wood, at $$7\frac{1}{2}$ a cord; how many cords did it take?

23. If a frog should be 12 days in getting out of a well, by leaping up 12 feet every morning, and falling back 4 feet every evening, how deep is the well?

24. A student, having a bible, a dictionary, and an algebra upon his table, was asked the price of each ; he answered that his bible cost twice as much as his dictionary, the dictionary cost twice as much as the algebra, and that the three books cost $ 10 ; what was the cost of each book ?

LESSON LI.

1. If $ 6 worth of provisions will last 5 men 7 days, how long will they last 10 men ?

ANALYSIS BY RATIO. *5 men are* $\frac{5}{10}$, *or* $\frac{1}{2}$, *of* 10 *men, and the provisions which will last 5 men 7 days will last* 10 *men* $\frac{1}{2}$ *of 7 days, or* $3\frac{1}{2}$ *days.*

ANALYSIS BY UNITY. — *The provisions which will last 5 men 7 days, will last* 1 *man 5 times 7 days, or 35 days ; and what will last 1 man 35 days, will last 10 men* $\frac{1}{10}$ *of 35 days, or* $3\frac{1}{2}$ *days.*

2. If a barrel of meat will last 12 men 20 days, how long will it last 16 men ?   36 men ?   40 men ?

3. If a barrel of beer last 5 men 16 days, how long will it last 8 men ?   How many men will it last 8 days ?

4. If a pole 40 feet high cast a shadow of 25 feet, how long a shadow will be cast at the same time by a similar pole 16 feet high ?   How high a pole will cast a shadow of 30 feet ?

5. If I borrow $ 20 for 3 months, how many dollars can I lend for 5 months, in order to return the favor ?   For how many months can I lend $ 30 ?

6. If a man can do a certain piece of work in 5 days, by laboring 8 hours a day, how many hours a day must he labor in order to do it in 4 days ?   In how many days would he do it by laboring 6 hours a day ?

7. If 8 bushels of grain will last 7 horses 5 days, how long will 16 bushels last 4 horses?

ANALYSIS BY RATIO. — *If 8 bushels of grain will last 7 horses 5 days, 16 bushels will last the same number of horses 2 times 5 days, or 10 days; and if they will last 7 horses 10 days, they will last 4 horses ¼ of 10 days, or 17½ days.*

ANALYSIS BY UNITY. — *If 8 bushels of grain will last 7 horses 5 days, 8 bushels will last 1 horse 7 times 5 days, or 35 days; 1 bushel will last 1 horse ⅛ of 35 days, or 4⅜ days; 16 bushels will last 1 horse 16 times 4⅜ days, or 70 days; and 16 bushels will last 4 horses ¼ of 70 days, or 17½ days.*

8. If a ton of hay will last 8 cows 7 days, how long will it last 11 cows?

9. If a ton of a hay will last 7 cows 8 days, how many cows will it last 14 days?

10. If $300 gain $12 in 8 months, what sum would it require to gain $8 in 2 months?

11. If $100 gain $6 in 12 months, how many months would it require for $400 to gain $10?

12. If 40 bushels of oats are sufficient for 5 horses 6 weeks, how many bushels would it require to supply 15 horses 8 weeks?

13. If 20 oxen eat 4 tons of hay in 30 days, how many oxen would it take to eat 12 tons in 60 days? In 50 days? In 75 days?

14. If 7 men can build 8 rods of wall in 4 days, how many days would it take 14 men to build 32 rods of wall? 40 rods? 80 rods?

15. If 12 men can reap a field of 4 acres in 8 days, by laboring 6 hours a day, how many acres would 6 men reap in 12 days, by laboring 7 hours a day?

16. If the interest of $250 for 10 months is $12½, what is the interest of $450 for 5 months?

17. If 2 oxen, or 3 cows, will eat 3 tons of hay in 18 weeks, how much hay will 6 oxen and 1 cow eat *in 9 weeks?*

18.   There are in a fort 200 men, with provisions sufficient to last 6 months; how many must leave after the provisions are half gone, that the remaining men shall have just sufficient for 6 months?

19.   If 8 men can dig a ditch 30 rods long in 20 days, how long will it take 10 men to dig a ditch 15 rods long?

20.   A workman, laboring 10 hours a day, will build in 20 days 30 rods of wall; how long will it take 10 workmen, laboring 9 hours a day, to build 45 rods?

21.   If 6 men can do the work of 24 women, and 4 women do the work of 6 boys, how many men can do the work of 18 boys?

ANALYSIS BY RATIO. — *Since 6 boys do the work of 4 women, 18 boys will do the work of 3 times 4 women, or 12 women; and as 24 women do the work of 6 men, 12 women, or 18 boys, will do the work of $\frac{1}{2}$ of 6 men, or 3 men.*

NOTE. — In analyzing by unity, we may either find that 1 man can do the work of 6 boys, or that 1 boy will do $\frac{1}{6}$ as much as a man.

22.   If 5 pounds of cheese are equal in value to 2 pounds of butter, and 6 pounds of butter to 2 bushels of corn, how many pounds of cheese will pay for 4 bushels of corn?

23.   If the relative value of oak wood to spruce is as 2 to 1, and that of spruce to pine as 7 to 8, how many cords, composed of spruce and pine in equal parts, will equal 10 cords of oak?

24.   If 3 men can build a boat in 12 days, when the days are 12 hours long, how long will it take 5 men to build the same, when the days are 10 hours long?

25.   If $150 gain $9 in 12 months, in what time will $200 gain $18?

26.   When 2\frac{1}{4}$ will purchase $\frac{3}{4}$ of a barrel of

flour, what part of a barrel can be purchased for
$4½?

27.   If 2¾ bushels of potatoes cost $1⅓, how much
will ⅔ of a bushel cost?

28.   Two kinds of writing-books have the same
number of pages, but the first has 12 lines to a page,
and the second 16; how many books of the second
kind will contain as many lines as 1 dozen books of
the first kind?

ANALYSIS BY UNITY.   *12 books with* 12 *lines to a page,*
would contain as many lines as 12 *times* 12 *books, or* 144
books, with 1 *line to a page ; and* 144 *books, with* 1 *line to a*
page, would contain as many lines as $\frac{1}{16}$ *of* 144 *books, or* 9
books, with 16 *lines to a page.*

29.   I can perform a journey in 10 days by travel-
ing at the rate of 4 miles an hour; how can I per-
form it in 8 days, if I travel the same number of
hours a day?

30.   If 4 pounds of flour will make 40 four-cent
loaves of bread, how many six-cent loaves can be
made from the same quantity?

LESSON LII.

1   If 12 yards of cotton cloth cost $1.60, how
much will 39 yards cost?

NOTE.   39 yards = 3¼ times 12 yards.

2.   If 7 horses consume 16 tons of hay a year,
how many tons do 5 horses consume?

3.   If 12 men can dig a well in 4 days, in how
many days can 15 men dig it?

4.   11 is to 33 as $0.25 is to what sum?

5.   If 3 and 4 were 12, what, on the same sup-
position, would 2 and 3¼ be?

6. If the difference of two numbers is 30, and the greater is 200, what is the less?

7. When the difference between two numbers, and the greater number are given, how do you find the less?

8. If the difference of two numbers is 30, and the less 130, what is the greater?

9. When the difference between two numbers, and the less number are given, how do you find the greater?

10. Bought 240 pounds of beef, at 11 cents a pound, and sold half of it for 13 cents, and half for 14 cents a pound; how much did I gain?

11. Bought 25 yards of broadcloth, at $6 per yard, and paid for the same in wood, at $7 per cord; how much did it take?

12. What cost 96 bushels of potatoes, at 25 cents a bushel?

NOTE. 25 cents is ¼ of a dollar.

13. If the sum of two numbers is 5, and their difference is 3, what are the numbers?

ANALYSIS. — *As the greater number is 3 more than the less, their sum, or 5, must be 3 more than twice the less; hence twice the less must be 5 — 3, or 2, and the less number must be ½ of 2, or 1. The greater number is 1 + 3, or 4.*

NOTE. — In the same way, 5 is 3 less than twice the greater, and the greater must be ½ of 5 + 3, or 8. The sum of two numbers added to their difference always gives twice the greater, and their difference taken from their sum gives twice the less.

14. The sum of two numbers is 17, and their difference 5; what are they?

15. Divide 65 into two parts, one of which shall be 19 greater than the other.

16. Two boys, on counting their money, found that the one had $46 more than the other, and together they had $54; how much had they each?

17. The sum of two numbers is $7\frac{1}{2}$, and their difference $2\frac{1}{4}$; what are they?

18. If 25 barrels of flour cost $$131\frac{1}{4}$, how much will $3\frac{1}{8}$ barrels cost?

19. What cost $\frac{7}{8}$ of a hogshead of molasses, at $\frac{3}{8}$ of a dollar a gallon?

20. What number, increased by $\frac{1}{4}$, by $\frac{1}{2}$, and by $\frac{1}{6}$ of itself, will amount to 39?

21. A and B undertake to travel round a circular island 20 miles in circuit, both starting from the same point, and going round in the same direction; when A has traveled 19 miles, and B 8, how far apart are they?

22. Divide $100 among A, B, and C, so that B shall have $20 more than A, and C $15 less than B.

Note. — C has $5 more than A, and $100 must be $25 more than 3 times A's share.

23. A man who has 60 sheep and lambs, finds that the number of the sheep, less the number of lambs, is $\frac{2}{5}$ of the flock; how many are there of each kind?

24. What part of the principal is $7\frac{1}{2}$ per cent. interest?

25. At what per cent. interest will $10 become $15, in 5 years?

26. The difference of two numbers is 5, and the less number is $\frac{5}{6}$ of the greater; what are the numbers?

27. When an article is sold at $\frac{2}{3}$ of its cost, what is the loss per cent.?

28. John Smith gave 9 cents a dozen for apples, and had 12 cents left; but, had he paid 12 cents a dozen, he would have spent all his money; how many apples did he buy?

Note. — The 12 cents left was how many cents a dozen?

29. A man bought a certain number of quires of paper, at 18 cents a quire, and had 84 cents left; had he bought just as many quires at 24 cents a quire, he would have had 6 cents left; how much money had he?

30. Bought rice at 6½ cents a pound, and sold it at 8⅛ cents a pound; what per cent. was the gain?

31. A laborer agreed to work 20 days upon the condition that for every day he worked he should receive $1.50, but for every day he was idle he should forfeit 50 cents; he received $18; how many days did he work?

ANALYSIS. — *Had he worked all the time, he would have received 20 times $1½, or $30; he therefore lost $30 — $18, or $12, by being idle. For each day that he was idle he lost $1½ + $½, or $2, and he must have been idle as many days as $2 are contained times in $12, which are 6. Therefore he was idle 6 days, and worked 20 days — 6 days, or 14 days.*

NOTE. — The first form of division may be avoided thus: He lost $1 in ½ of a day, and $12 in 12 times ½ of a day, or 6 days.

32. A laborer agreed to work 30 days for $2 a day, and for every day he was idle to pay ¼ of a dollar for his board; he received $38; how many days was he idle?

33. A gentleman bought 15 cords of wood, oak and pine, for $55; he paid $5 a cord for the oak, and $3 a cord for the pine; how many cords were there of each, and how much did each kind cost?

34. George is 2 years older than James, and 3 years younger than Albert, but Samuel's age is equal to the sum of George's and James's; the sum of all their ages is 29 years; how old is each?

LESSON LIII.

1. How many yards of cambric which is ¾ of a yard wide, will be required to line 30 yards of cloth that is 1¾ yards wide?

2. In a certain school ⅛ of the pupils study algebra, ¼ geometry, and the remainder arithmetic; what per cent. of the whole are in each of the studies named?

3. ¼ less ⅛, plus ¾, multiplied by 1½, is how many times ¼?

4. A dog one night killed 17 sheep, which were 5⅔ per cent. of a flock; how many of the flock were spared?

5. How many yards of cloth that is ¾ of a yard wide, are equal to 9 yards that is ½ of a yard wide?

6. Joseph earned $1⅛ a day, and spent $3.50 a week for board, and ⅛ of the remainder of his earnings for clothes; at that rate, how much can he save in 4 weeks?

7. If two men together can mow a field in 10 days, and one of them alone can mow it in 15 days, in what time can the other mow it?

8. The head of a fish is 9 inches long; the tail is as long as the head and half the body, and the body is as long as the head and tail both; how long is the fish?

ANALYSIS. — *By the conditions of the problem, the length of the head is 9 inches, the length of the tail is 9 inches + ½ the length of the body, and the length of the head and tail, or the length of the body, is 18 inches + ½ the length of the body; therefore ⅔ — ½, or ½, the length of the body is 18 inches, &c.*

9. The head of a fish is 7 inches long; the tail is as long as the head and one third of the body,

and the body is as long as the head and tail both; how long is the fish?

10. In what time will a given principal double itself, at $5\frac{1}{2}$ per cent. interest?

11. A man can dig a ditch in 3 days, and his son in 5 days; in what time can they dig it together?

12. Jason has 25 per cent. more money than Edward; what per cent. less has Edward than Jason?

13. In how long a time will $60 gain $6.80, at 7 per cent. interest?

14. How much water must be mixed with 10 gallons of brandy, worth $8 a gallon, that the mixture may be worth only $7 a gallon?

15. A and B set out to travel round a certain island which is 20 miles in circuit; A travels at the rate of 5 miles an hour, and B 7 miles an hour; how long will it take B to overtake A?

16. Bought a watch, chain, and key for $125; the watch cost 5 times as much as the chain, and the key cost 95 per cent. less than the watch; what was the cost of each?

17. If $100 have been borrowed at 6 per cent. interest, for 1 year, how long must $250 be loaned at the same rate per cent., to requite the favor?

18. At what per cent. interest will $30 become $60 in $12\frac{1}{2}$ years?

19. If you should purchase a lot of sheep, at $2.50 apiece, and should lose $\frac{1}{4}$ of the number, at what price each must you sell the rest so as neither to gain nor lose?

20. A man left $\frac{1}{3}$ of his estate to his wife, $\frac{1}{4}$ of the remainder to his son, and $\frac{2}{4}$ of the remainder to his daughter; the three legacies amounted to $900; how much was the whole estate?

21. A lady having a number of peaches, gave

away ¼ of them, and ⅔ of the remainder, and had 27 left; how many had she?

22. Four men rent a field for $16; A puts in 6 cows, B 8 cows, C 4 cows, and D as many cows as his paying $\frac{1}{10}$ of the rent entitles him to; what part of the rent did A, B, and C each pay, and how many cows did D put in?

23. Sold a watch for $35, and thereby lost 25 per cent., when there ought to have been gained 30 per cent.; how much was it sold below its proper value?

24. Sold a cart for $30, and thereby lost 20 per cent.; at what price should it have been sold to have gained 20 per cent.?

25. If 4 horses consume 21 bushels of grain in 6 days, how many bushels will 8 horses consume in 12 days?

26. A circular garden is 3 rods in diameter; what is its circumference?

NOTE. — The circumference of a circle, or the distance round it, is 3¹⁄₇, nearly, of its diameter, or the distance across it through the center.

27. A certain circular pond is 11 miles in circumference; what is its diameter?

28. If 4½ bushels of wheat cost 9\frac{9}{10}$, what cost ⅔ of a bushel?

29. If 3 men can mow a field in 10 hours, how long will it take them, if 2 men be added to their number?

30. The circumference of a circle is 22 inches; what is its area?

NOTE. — The area of a circle is equal to half the diameter multiplied by half the circumference.

31. How much greater area has a circle 22 inches in circumference, than a square of the same perimeter?

NOTE. — The perimeter of the square is the distance round it.

32. If a flagstaff 30 feet in hight, at a certain hour, casts a shadow of 20 feet, what must be the hight of that staff which, at the same time, casts a shadow of 25 feet?

33. Bought 60 apples at 5 for 2 cents, and sold half of them at 2 for a cent, and half at 3 for a cent; how much was the gain?

34. John has 26 cents' worth of marbles, $\frac{2}{5}$ of the number of which are worth 8 to a cent, $\frac{3}{10}$ 2 to a cent, and the remainder 1 cent apiece; how many has he?

35. I went to the city with $5\frac{1}{2}$ in my purse; I spent $\frac{1}{10}$ of it in paying my fare, $\frac{1}{8}$ of the remainder for a reading-book, and bought with what was left twice as many grammars at 20 cents apiece, as I did spelling-books at 15 cents apiece; how many did I buy of each?

LESSON LIV.

1. The sum of two numbers is 19, and twice the first added to 5 times the second is 74; what are the numbers?

Note. — Twice the first added to twice the second is 38; hence 3 times the second is 36.

2. Williams and Brown enter into business together; Williams puts in 4 times $\frac{1}{5}$ of what Brown does; they both put in $1680, and gain $700; what is each one's share of the gain?

3. A barrel of flour and a cord of wood cost $17, and 2 barrels of flour cost $10 more than a cord of wood; how much does each cost?

4. A hare starts 25 leaps in advance of a hound, and takes 4 leaps to the hound's 3; but 2 of the

hound's leaps equal 3 of the hare's; how many leaps must the hound take to overtake the hare?

5. An apprentice receives from his employer 5 cents for every good photograph which he produces, and forfeits 10 cents for the material consumed in every one he spoils; after making 50 attempts, he received 1 dollar; how many did he spoil?

6. Two men have a flock of sheep; A has 15 more than half of the number that B has, and both have 54; how many have each?

7. Smith and Robinson go into partnership, each putting in $300; afterwards Smith puts in twice as much, and Robinson 3 times as much, as before; what share of the capital has each contributed?

8. The sum of two numbers is 24, and 3 times the first, less twice the second, is 17; what is each?

9. I have one book which has 16 pages to a sheet, and another which has 36; they have each the same number of pages, and together 39 sheets; how many pages has each?

10. If $\frac{2}{3}$ of the time to noon is equal to $\frac{5}{6}$ of the time past midnight, what is the hour?

ANALYSIS. — *If $\frac{2}{3}$ of the time to noon is equal to $\frac{5}{6}$ of the time past midnight, $\frac{1}{3}$ of the time to noon is equal to $\frac{1}{2}$ of $\frac{5}{6}$, or $\frac{5}{12}$, of the time past midnight, and $\frac{3}{3}$, or the time to noon, is equal to 3 times $\frac{5}{12}$, or $\frac{5}{4}$, of the time past midnight; hence, as the time past midnight is $\frac{4}{4}$ of itself, the time from midnight to noon, or 12 hours. is $\frac{5}{4} + \frac{4}{4}$, or $\frac{9}{4}$, of the time past midnight, and the time past midnight must be $\frac{4}{9}$ of 12 hours, or $5\frac{1}{3}$ hours. Therefore the required time is 20 minutes past 5 o'clock in the morning.*

11. A man being asked the time of day, answered that 8 times the time to noon was equal to twice the time to midnight; what time was it?

NOTE. 12 hours is here the *difference* of the times, instead of their *sum*.

12. A man being asked what time it was, answered that the time past noon was $\frac{1}{4}$ of the time past midnight; what time was it? .

13. A man being asked the time of day, answered that the time past noon was equal to $\frac{4}{5}$ of the time past midnight; what time was it?

14. A man said that $\frac{1}{4}$ of the time past noon was equal to $\frac{1}{6}$ of the time to midnight; what was the hour?

15. A and B start from the same point, and travel in the same direction around a square, each side of which measures 5 miles; A travels at the rate of $4\frac{1}{2}$ miles an hour, and B at the rate of 8 miles an hour; in what time will they be together again? How many miles will each have traveled? How many times will each have been around the square?

16. There is a cask containing 50 gallons of wine; if one half of the wine be drawn off and an equal quantity of water be added, and then one fifth of this mixture be drawn off and the same quantity of water poured in, how many gallons of wine and how many of water are there in the cask?

17. A cask capable of holding 75 gallons, contains 50 gallons of wine; if enough water be poured in to fill the cask and one third of the mixture be drawn off, and then 10 gallons of water be poured in and one sixth of the mixture drawn off, how many gallons of wine and how many of water remain in the cask?

18. A wolf can eat a sheep in $2\frac{1}{2}$ days, a hound can eat it in $3\frac{3}{4}$ days, and a mastiff in 4 days; after the wolf has eaten $\frac{1}{2}$ of a day and the hound $\frac{1}{4}$ of a day, how long will it take the hound and mastiff together to eat what remains?

19. A bought a horse for 25 per cent. less than

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his real value, and sold him to B for 25 per cent. more than his value: how much per cent. did A make on his purchase ?   How much per cent. would B lose were he to sell the horse for the same price that A gave ?

20.   A lady has 2 silver cups, and but one cover for both; the cover weighs 10 ounces; now, if the cover be put on the first cup, it will make the weight double that of the second, and if the cover be put on the second, it will make the weight triple that of the first: what is the weight of each ?

21.   There is a bin containing 30 bushels of wheat; if I take from it 10 bushels and add 10 bushels of rye, then take away 10 bushels of the mixture and add 10 bushels more of rye, then take away 10 bushels of this mixture and add 10 more of rye, supposing the wheat and rye to have been thoroughly mixed each time, how many bushels of wheat, and how many of rye would the bin contain ?

22.   If I should each time add the 10 bushels of rye *before* taking out the 10 bushels of the mixture, and perform the operation three times, as in the last example, how many bushels of each would remain in the bin ?

23.   If a merchant purchases goods for cash to the amount of $500, when money is worth 2 per cent. a month, what sum will he gain by selling the goods at the end of 4 months, but on 3 months' credit, at an advance of 20 per cent. upon the cost ?

24.   A father said to his son, " 4 years ago I was 8 times as old as you, but 8 years hence I shall be 2 times as old as you"; what was the age of each ?

25.   A gentleman let ¼ of his money at 5 per cent., and the remainder at 6 per cent.; the interest amounted to $90; what were the sums let ?

26. In 1854 James was 10 times as old as William, but in 1862 he was only twice as old; how old is each of them the present year?

27. A has 50 per cent. more property than B, and B has 50 per cent. more than C; how much per cent. more has A than C? How much per cent. less has C than A?

28. A man, being asked the time of day, answered that $\frac{3}{4}$ of the time past midnight was equal to $\frac{2}{4}$ of the time past noon; what was the hour?

29. If you divide $27.50 between A and B, giving A 2\frac{1}{2}$ as often as you give B 6\frac{2}{3}$, how much will each receive?

30. A cistern has 3 pipes; the first will fill it in 1$\frac{1}{3}$ hours, the second will fill it in 2 hours, and the third will empty it in 1$\frac{1}{2}$ hours; if the pipes are all open at the same time, how long will it take to fill the cistern?

31. If an army march 150 miles in 7$\frac{1}{2}$ days, by marching 8 hours a day, how far should it march in 5$\frac{1}{4}$ days, by marching 10 hours a day?

32. The sum of two numbers is 5$\frac{3}{8}$, and their difference 1$\frac{1}{8}$; what are they?

33. After spending $\frac{3}{4}$ of my money, I found if I had spent 1\frac{3}{4}$ less, I should have spent just $\frac{1}{4}$ of my money; how much did I have?

34. A grocer has two kinds of tea, one of which is worth 40 cents a pound, and the other 50 cents a pound; how many pounds of each must be taken to form a chest of 40 pounds, which shall be worth 44 cents a pound?

35. The sum of three numbers is 18, the sum of the first and second is equal to the third, and half the sum of the first and third is equal to the second; what are the numbers?

36. A farmer employed 8 men and 8 boys one

day for $5, and another day, at the same wages, 4
men and 6 boys for $8; what was the daily wages
of each?

37. What number is that to which if 3 and 14
be separately added, the first sum will be $\frac{1}{2}$ of the
second?

38. Three men, A, B, and C, each have a sum of
money in their pockets; A has $3, A and C together
have 3 times as much as B, and B and C together
have 11 times as much as A; how much have B and
C, respectively?

39. Smith, Jones, and Brown each have a sum of
money at interest at 5 per cent., and these sums
are to each other as $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, respectively; the
annual income of the three sums taken together is
$90; what is the principal that each one has at
interest?

40. Three men hired a pasture for $76; at first
A put in 4 horses and 8 cows, B 6 horses and 12
sheep; afterwards, when the grass was half eaten up,
C put in 24 sheep; now, supposing every horse eats,
in a given time, as much as 4 sheep, and every cow
as much as 3 sheep, what ought each man to pay?

41. Robinson, Savage, and Harrison agree to-
gether to do a piece of work; they are to receive for
it $200, to be divided in the proportion of 5, 4, and
3, respectively, for the same amount of work; but
Robinson, whose labor is worth most, is absent $\frac{1}{2}$ of
the time, and Harrison, whose labor is worth least, is
absent $\frac{1}{3}$ of the time; how should the money be
justly divided among them?

APPENDIX.

EXERCISES INVOLVING LARGE NUMBERS.

LESSON LV.

This and the following Lessons are intended only for advanced classes, and may be omitted at the option of the teacher.

1. What is the sum of 38 and 46 ?

SOLUTION. 38 *is* 3 *tens and* 8 ; 46 *is* 4 *tens and* 6 ; 38 *and* 46 *are then* 3 *tens and* 4 *tens, or* 7 *tens, and* 8 *and* 6, *or* 14 ; 14 *is* 1 *ten and* 4 ; *therefore* 38 *and* 46 *are* 7 *tens and* 1 *ten and* 4, *which are* 8 *tens and* 4, *or* 84.

2. What is the sum of 23 and 25 ? 29 and 33 ?
3. What is the sum of 57 and 34 ? 76 and 18 ?
4. How many are 37 and 39 ? 62 and 28 ?
5. How many are 51 and 27 ? 26 and 67 ?
6. How many are 68 and 54 ?

NOTE. 6 tens and 5 tens are 11 tens; 8 and 4 are 12, or 1 ten and 2; 11 tens and 1 ten and 2 are 12 tens and 2, or 122.

7. How many are 66 and 67 ? 58 and 45 ?
8. How many are 93 and 14 ? 56 and 65 ?
9. How many are 83 and 36 ? 68 and 12 ?
10. How many are 95 and 21 ? 70 and 25 ?
11. How many are 54 and 13 ? 28 and 42 ?
12. How many are 84 and 74 ? 21 and 62 ?
13. How many are 13 and 63 ? 45 and 15 ?
14. How many are 98 and 22 ? 36 and 43 ?
15. How many are 53 and 63 ? 75 and 28 ?

16. How many are 28 and 31 ? 64 and · 39 ?
17. How many are 91 and 26 ? 67 a...u 51 ?
18. How many are 84 and 45 ? 77 and 33 ?
19. How many are 28 and 43 ? 27 and 84 ?
20. How many are 63 and 19 ? 36 and 72 ?
21. How many are 47 and 38 ? 75 and 22 ?
22. How many are 54 and 68 ? 36 and 25 ?
23. How many are 95 and 16 ? 85 and 21 ?
24. How many are 101 and 17 ? 92 and 35 ?
25. How many are 203 and 37 ? 95 and 35 ?
26. How many are 180 and 25 ? 309 and 90 ?
27. How many are 68 and 34 ? 27 and 126 ?
28. How many are 58 and 95 ? 137 and 101 ?
29. How many are 154 and 44 ? 106 and 94 ?
30. How many are 130 and 67 ? 219 and 31 ?

LESSON LVI.

1. How many are 60, 17, and 2 ?
2. How many are 25, 72, and 3 ?
3. How many are 44, 32, and 7 ?
4. How many are 13, 45, and 61 ?
5. How many are 57, 6, and 43 ?
6. How many are 16, 93, and 12 ?
7. How many are 99, 40, and 5 ?
8. How many are 70, 56, and 8 ?
9. How many are 20, 50, and 13 ?
10. How many are 27, 38, and 30 ?
11. How many are 50, 41, and 11 ?
12. How many are 15, 27, and 70 ?
13. What is the sum of 19, 75, and 15 ?
14. What is the sum of 68, 49, and 1 ?
15. What is the sum of 60, 35, 6, and 5 ?
16. How many are 50, 40, 7, and 3 ?
17. How many are 65, 25, 10, and 7 ?

18. How many are 70, 37, 13, and 6?
19. How many are 61, 17, 32, and 9?
20. How many are 106, 140, 7, and 11?
21. How many are 300, 130, 76, and 5?
22. How many are 70, 90, 16, and 87?
23. How many are 63, 17, 31, 10, and 8?
24. How many are 110, 52, 18, 11, and 4?
25. How many are 440, 64, 34, and 12?
26. How many are 2000, 513, 7, and 80?

LESSON LVII.

1. 33 taken from 76 leave how many?

SOLUTION. 76 *is* 7 *tens and* 6; 33 *is* 3 *tens and* 8; 3 *tens from* 7 *tens leave* 4 *tens, and* 3 *from* 6 *leave* 8; *therefore* 33 *from* 76 *leave* 4 *tens and* 3, *or* 43.

2. 28 taken from 99 leave how many?
3. Take 33 from 85; 46 from 78.
4. Take 54 from 97; 5 from 98.
5. 42 less 8 are how many?

NOTE. 8 cannot be taken from 2, but we may take 1 ten from the 4 tens of the 42, and adding it to the 2, have a number from which 8 can be taken. Then, 8 from 42 will be the same as 3 tens, and 1 ten and 2, less 8; 1 ten and 2, or 12, less 8, are 4; therefore 42 less 8 are 3 tens and 4, or 34.

6. From 53 take 6; take 5; take 9.
7. Take 1 from 20; 2 from 30; 3 from 40.
8. Take 4 from 50; 5 from 60; 7 from 70.
9. Take 6 from 80; 7 from 90; 9 from 100.
10. Take 8 from 85; 6 from 74; 9 from 95.
11. Take 28 from 76.

NOTE. 8 cannot be taken from 6, but 76 may be called 6 tens and 16; 8 from 16 leave 8, and 2 tens from 6 tens leave 4 tens; therefore 28 from 76 leave 4 tens and 8, or 48.

12. Take 37 from 100; 29 from 73; 56 from 128.

13. Take 39 from 143; 91 from 180; 37 from 147.

14. Take 105 from 165; 77 from 296; 42 from 163.

15. Take 110 from 585; 38 from 408; 125 from 260.

16. Take 500 from 1001; 350 from 940; 662 from 893.

LESSON LVIII.

1. How many are 3 times 18?

SOLUTION. 18 *is* 1 *ten and* 8; 3 *times* 1 *ten are* 3 *tens*; 3 *times* 8 *are* 24, *or* 2 *tens and* 4; *and* 3 *tens and* 2 *tens and* 4 *are* 5 *tens and* 4, *or* 54; *therefore* 3 *times* 18 *are* 54.

2. How many are 5 times 16? 6 times 17?

3. How many are 8 times 14? 8 times 15?

4. How many are 8 times 13?

NOTE. 8 times 1 ten are 8 tens; 8 times 3 are 2 tens and 4; 8 tens and 2 tens and 4, are 10 tens and 4, or 104.

5. How many are 7 times 18? 8 times 19?

6. How many are 6 times 33? 2 times 67?

7. How many are 7 times 36?

NOTE. 7 times 3 tens are 21 tens; 7 times 6 are 4 tens and 2; 21 tens are 2 hundreds and 1 ten; 2 hundreds and 1 ten and 4 tens and 2, are 2 hundreds, 5 tens and 2, or 252.

8. How many are 9 times 38? 7 times 76?

9. How many are 9 times 37? 8 times 29?

10. How many are 6 times 49? 7 times 49?

11. How many are 9 times 19? 7 times 55?

12. How many are 4 times 48? 3 times 36?

13. How many are 2 times 85? 4 times 44?

14. How many are 6 times 29? 7 times 24?

15. How many are 9 times 39? 8 times 44? 7 times 51?

16. How many are 6 times 61? 5 times 74? 4 times 93?

17. How many are 5 times 77? 8 times 90?

LESSON LIX.

1. How many are 14 times 18 ?

SOLUTION. 14 *is* 1 *ten and* 4; 10 *times* 18 *are* 18 *tens, and* 4 *times* 18 *are* 4 *tens and* 3 *tens and* 2, *or* 7 *tens and* 2; 18 *tens and* 7 *tens and* 2 *are* 25 *tens and* 2, *or* 252.

2. How many are 15 times 16 ?   15 times 12 ?
3. How many are 16 times 26 ?   13 times 23 ?
4. How many are 13 times 17 ?   16 times 24 ?
5. How many are 11 times 31 ?   12 times 32 ?
6. How many are 18 times 19 ?   19 times 20 ?
7. How many are 20 times 21 ?   13 times 33 ?
8. Find the product of 23 and 17; 22 and 18; 21 and 19.
9. Find the product of 23 and 14; 29 and 13; 31 and 11.
10. Find the product of 28 and 16; 27 and 19; 26 and 20.
11. Find the product of 37 and 11; of 36 and 12; of 35 and 13.
12. How many are 11 times 44 ?   12 times 35 ? 13 times 32 ?
13. Multiply 11 by 18; 21 by 18; 31 by 18.
14. Multiply 12 by 17; 22 by 17; 32 by 17.
15. Multiply 13 by 14; 23 by 14; 33 by 14; 43 by 14.
16. Required the product of 17 by 17; of 27 by 17; of 37 by 17.
17. Required the product of 18 and 17; of 28 and 17; of 38 and 17.
18. Multiply 11 by 11; by 21; by 41.
19. Multiply 16 by 12; 18 by 13; 16 by 14; 28 by 14.
20. How many are 6 times 16 ?   16 times 16 ?

16 times 17? 16 times 18? 16 times 23? 18 times 25.

21. Multiply 29 by 27.

NOTE. 29 by 2 tens and 7 are 58 tens and 203, or 20 tens and 3; 58 tens, 20 tens, and 3 are 78 tens and 3, or 783.

22. Multiply 28 by 28; 30 by 20; 29 by 21; 27 by 27.

23. How many are 23 times 27? 24 times 26? 25 times 25?

24. Multiply 27 and 18 together; 25 and 28; 24 and 29.

LESSON LX.

1. How many times is 8 contained in 88?

SOLUTION. 88 *is* 8 *tens and* 8; 8 *in* 8 *tens* 1 *ten time, and* 8 *in* 8 *units* 1 *unit time. Therefore* 8 *is contained in* 88, 1 *ten and* 1 *unit times, or* 11 *times.*

2. How many times 6 in 90?

NOTE. 6 in 9 tens 1 ten time, and 3 tens remaining; 3 tens remaining equal 30 units; and 6 in 30 units 5 units times. Therefore, 6 is in 90, 1 ten and 5 units times, or 15.

3. How many times 8 in 96?

NOTE. 8 in 9 tens 1 ten times, and 1 ten remaining; 1 ten remaining equals 10 units; 10 units and 6 units are 16 units; and 8 in 16 units 2 units times. Therefore 8 is in 96, 1 ten and 2 units times, or 12 times.

4. Divide 50 by 5; 60 by 5; 75 by 5; 85 by 5; 90 by 5.

5. Divide 66 by 6; 78 by 6; 84 by 6.

6. Divide 84 by 7; 98 by 7; 77 by 7.

7. Divide 99 by 9; 63 by 7; 88 by 4.

8. 126 contains 6 how many times?

NOTE. 126 equal 12 tens and 6 units.

9. How many times will 126 contain 7 ? Divide 133 by 7 ; 184 by 8 ; 224 by 8.

10. Divide 252 by 3 ; by 4 ; by 6 ; by 7 ; by 9.

11. How many quarts of milk, at 6 cents a quart, can you buy for 9 dimes and 6 cents? How many for 2 dollars, 2 dimes, 2 cents?

12. How many times will 129 contain 8 ?

ANS. 16¼ *times.*

13. Divide 169 by 7 ; 168 by 7 ; 168 by 8.

14. How many times is 7 contained in 169? In 172? In 175 ?

15. Divide 868 by 8; 808 by 8; 848 by 8; 888 by 8.

NOTE. 80 tens divided by 8, gives 10 tens, or 1 hundred.

16. Divide 896 by 7.

NOTE. — We find the *hundreds* in the quotient first; 7 is contained in 8 hundreds 1 hundred times, and 1 hundred over.

17. Divide 968 by 8 ; 1048 by 8 ; 1136 by 8.

18. How many times 11 in 121 ?

SOLUTION. 121 *is* 1 *hundred,* 2 *tens, and* 1 *unit ;* 11 *is not contained in the* 1 *at the left ; but* 1 *hundred is* 10 *tens, and with the* 2 *tens make* 12 *tens, in which* 11 *goes* 1 *ten time with* 1 *ten remaining ;* 1 *ten remaining equals* 10 *units ;* 10 *units and* 1 *unit are* 11 *units ; and* 11 *in* 11 *units* 1 *unit time. Therefore* 11 *is in* 121, 1 *ten and* 1 *unit, or* 11 *times.*

19. Divide 221 by 11; 143 by 11.

20. Divide 144 by 12; 168 by 12; 180 by 12.

21. Divide 221 by 13; 216 by 18; 225 by 15.

LESSON LXI.

The SQUARE of any number is the product of that number by itself.

1. What is the square of 11 ?

SOLUTION. 11 *is* 1 *ten and* 1; 1 *ten time* 11 *is* 110 ; 1

time 11 *is* 11; *and* 110 *and* 11 *are* 121. *Therefore the square of* 11, *or the product of* 11 *by itself, is* 121.

2. What is the square of 14 ? Of 13 ? Of 16?

3. What is the square of 12 ? Of 18 ? Of 17?

4. What is the square of 25 ?

NOTE. — The square of any number whose unit figure is 5 is equal to *the product of the number denoting the tens by itself in-creased by* 1, *with the square of the units annexed.*

Thus, 25 is 2 tens and 5 units; 2 by 2 increased by 1, or 2 by 3, equal 6; 5 squared equals 25; and 6 with 25 annexed equals 625, the square of 25.

5. Required the square of 35 ? Of 45 ?

6. Required the square of 55 ? Of 65 ?

7. What is the square of 75 ? Of 85 ?

8. What is the square of 32 ?

NOTE. — The square of any number is equal to *the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

Thus, 32 is 3 tens and 2 units; 3 tens, or 30, squared is 900: twice 30 by 2 units is 120; the square of 2 units is 4 ; and 900 plus 120 plus 4 is 1024, the square of 32.

9. What is the square of 37 ? Of 56 ?

10. What is the square of 58 ? Of 73 ?

11. Required the product of 25 by 15.

NOTE. — The product of two unequal numbers, whose sum and difference are even numbers, is equal to *the square of half their sum, less the square of half their difference.*

Thus, 25 plus 15 equal 40 ; 25 less 15 equal 10 ; half of 40 equals 20 ; half of 10 equals 5 ; 20 squared is 400 ; 5 squared is 25 ; and 400 less 25 is 375, the product of 25 by 15.

12. What is the product of 16 by 26 ? 22 by 32?

13. What is the product of 39 by 27 ? 48 by 32?

14. What is the product of 61 by 13 ? 45 by 17?

LESSON LXII.

1. What is the square of $5\frac{1}{2}$?

NOTE. — The square of any mixed number, whose fractional part is $\frac{1}{2}$, is equal to *the product of the whole number by the next larger whole number, plus the square of the fraction.*

Thus, 5 by 6 equal 30 ; $\frac{1}{2}$ squared is $\frac{1}{4}$; and 30 plus $\frac{1}{4}$ equal $30\frac{1}{4}$, the square of $5\frac{1}{2}$.

2. What is the square of $4\frac{1}{2}$? Of $3\frac{1}{2}$? Of $6\frac{1}{2}$?

3. What is the square of $7\frac{1}{2}$? Of $10\frac{1}{2}$? Of $11\frac{1}{2}$?

4. What is the square of $4\frac{1}{4}$?

NOTE. — The square of any mixed number whose fractional part is $\frac{1}{4}$ is equal to *the square of the whole number plus half of itself, plus the square of the fraction.*

Thus, 4 squared is 16 ; half of 4 is 2 ; $\frac{1}{4}$ squared is $\frac{1}{16}$; and 16, plus 2, plus $\frac{1}{16}$ equal $18\frac{1}{16}$, the square of $4\frac{1}{4}$.

5. Required the square of $7\frac{1}{4}$? Of $8\frac{1}{4}$? Of $6\frac{1}{4}$?

6. What is the square of $10\frac{1}{4}$? Of $12\frac{1}{4}$? Of $9\frac{1}{4}$?

7. What is the product of $6\frac{1}{2}$ by $4\frac{1}{2}$?

NOTE. — The product of two unequal mixed numbers, when the fractional part of each is $\frac{1}{2}$, is equal to *the product of the whole numbers, plus half of the sum of the whole numbers, plus the product of the fractions.*

Thus, 6 by 4 equal 24 ; half of 6 plus 4 equals 5 , $\frac{1}{2}$ by $\frac{1}{2}$ equals $\frac{1}{4}$; and 24, plus 5, plus $\frac{1}{4}$, equal $29\frac{1}{4}$, the product of $6\frac{1}{2}$ by $4\frac{1}{2}$.

8. What is the product of $7\frac{1}{2}$ by $8\frac{1}{2}$?

9. What is the product of $10\frac{1}{2}$ by $6\frac{1}{2}$?

10. What is the product of $12\frac{1}{2}$ by $10\frac{1}{2}$?

11. What is the product of $4\frac{1}{8}$ by $4\frac{7}{8}$?

NOTE. — The product of two mixed numbers, having the same whole numbers, and the sum of the fractions 1, is equal to *the product of one of the whole numbers increased by 1, by the other; and the product of the fractions.*

Thus, 4 plus 1, or 5, by 4 equal 20 ; $\frac{1}{4}$ by $\frac{3}{4}$ equals $\frac{3}{4}$; and 20 plus $\frac{3}{4}$ equal $20\frac{3}{4}$, the product of $4\frac{1}{4}$ by $4\frac{3}{4}$.

12. What is the product of $6\frac{1}{4}$ by $6\frac{3}{4}$? $10\frac{1}{5}$ by $10\frac{4}{5}$? $12\frac{1}{8}$ by $12\frac{5}{8}$? $16\frac{3}{7}$ by $16\frac{4}{7}$?

LESSON LXIII.

1. Sold 3 plows for 33 dollars, and 2 wagons for 120 dollars ; what was the amount of the whole ?

2. Bought a watch for 120 dollars, and sold it for 85 dollars ; what was the loss ?

3. If John travels 14 miles in 1 day, how far will he travel in 12 days ?

4. How many barrels of flour, at $9 a barrel, can be bought for $116 ?

5. Required the number of square rods in a rectangular lot whose equal sides are 24 rods ?

6. What is the cost of $9\frac{1}{4}$ pounds of sugar, at $9\frac{1}{4}$ cents per pound ?

7. What cost $7\frac{1}{2}$ pounds of rice, at $7\frac{1}{2}$ cents a pound ?

8. How many square yards in a plot of ground 25 feet square ?

9. How many dollars will $4\frac{1}{8}$ cords of wood come to, at $4\frac{1}{8}$ a cord ?

10. If 9 yards of cloth cost 54 dollars, what cost $6\frac{1}{2}$ yards ?

11. Required the square of $12\frac{1}{4}$. Of $16\frac{1}{4}$.

12. When beef is $16\frac{2}{3}$ cents a pound, how much must be paid for $16\frac{1}{8}$ pounds ?

13. What is the product of $11\frac{1}{4}$ by $11\frac{1}{4}$? $11\frac{1}{2}$ by $11\frac{1}{2}$?

14. What is the difference between the square of 15, and that of 10 plus that of 5 ?

15. What is the difference between the square of 25, and that of 5 plus that of 20 ?

16. If a horse trot $9\frac{1}{2}$ miles in an hour, how many miles can he trot in $9\frac{1}{2}$ hours ?

REMARKS UPON THE PRECEDING LESSONS.

LESSONS I. - X. — It should be observed that concrete numbers can be added or subtracted only when they are considered of the same kind, or denomination. · We can not add 5 pounds of beef and 2 yards of cloth, neither can we·add 5 horses and 2 dogs, unless we consider them both merely as animals. (See Ex. 18, Les. 1.)

LESSON VII. — In adding large numbers, the tens may be added separately, thus: —

Ex. 11. 3 tens and 2 tens are 5 tens, that is, 30 and 20 are 50 ; 4 and 6 are 10, which added to 50 make 60. Therefore 34 and 26 are 60.

The addition may also be performed thus: —

3 tens and 4, plus 2 tens, are 5 tens and 4, that is, 34 and 20 are 54 ; 54 and 6 are 60. Therefore 34 and 26 are 60.

LESSON VIII. Ex. 34. — As there is nothing from which to subtract the 8, we may separate 40 into 30 and 10 ; then 20 from 30 leave 10, and 8 from 10 leave 2 ; 10 and 2 are 12. Therefore 40 less 28 are 12.

The subtraction may also be performed thus: —

40 less 20 are 20, and 20 less 8 are 12. Therefore 40 less 28 are 12.

LESSON XI. — Multiplication may be regarded as simply the addition of any number of equal quantities. Thus, 4 times 3 means that four 3's are to be added together.

As the sum of two quantities is of the same denomination as the quantities themselves, it is evident that the product must be of the same denomination as the multiplicand, which represents the quantities added. As the multiplier only shows how many quantities are added together, it is evidently used as an abstract number.

The Analysis given is a syllogism, and as the conclusion is always based upon two premises, or propositions, there must be two here. The first states the general principle applicable to the example, while the second gives the particular condition imposed. One of the premises of a syllogism is often so well known that it is not considered necessary to state it. Such is the case with the first proposition of the full form, and the omission of that gives rise to the abbreviated form.

It might be well to use the full form at first, and afterwards, when the pupil is sufficiently familiar with it, to allow the abbreviated form to be employed most of the time. A formula which is used so constantly should be as brief as possible, provided it be at the same time accurate. This abbreviated form may be introduced by either *since, as, if,* or a similar conjunction, or otherwise varied to suit the taste of different teachers.

Ex. 39, 40. — The form of analysis changes somewhat here, for the *greater* the number of men, the *less* the time required to perform a given piece of work. The following formulas conform more fully to the original model than those given in the text, but the expressions are less natural, and, in the second one, the statements are transposed.

1 man will do a piece of work in 4 times as many days as 4 men; then, if 4 men can do it in 8 days, 1 man will do it in 4 times 8 days, which are 32 days.

To do a piece of work in 1 day will require 8 times as many men as to do it in 8 days; then, if to do it in 8 days requires 4 men, to do it in 1 day will require 8 times 4 men, which are 32 men.

LESSON XIII. Ex. 12. — The manner in which the pupil is to obtain the result of multiplying numbers larger than 12 will depend upon the preferences of the teacher. If he wishes the pupil to *commit to memory* the results, such a portion of the table at the end of this book may be learned as will conform to the plans of the teacher. If the pupil is to *reason out* the result each time, then the tens and units may be multiplied separately, thus: —

2 times 10 are 20, and 2 times 5 are 10; 20 and 10 are 30. Therefore 2 times 15 are 30.

LESSON XIV. — This form of analysis is to be used when the two given quantities are of the *same kind*, and accords with the first part of the definition of Division. The result of such a division must be an abstract number, 2, instead of 2 peaches; but the formula provides for this by stating that the *number* of peaches is *the same as* the number resulting from the division.

By changing the form of the questions which relate to cost and buying, we obtain a somewhat neater form of analysis, which may be used instead of that given in the text: —

Since 2 cents buy 1 peach, 4 cents will buy as many peaches as 2 cents are contained times in 4 cents, which are 2.

This formula may also be still more briefly expressed: —

4 cents will buy as many peaches as 2 cents, the cost of 1 peach, are contained times in 4 cents, which are 2.

The full form is given below, rather for the sake of carrying out the analogy with other forms, than with the expectation that it will be much employed.

As many peaches can be bought for 4 cents as the cost of 1 peach is contained times in 4 cents; then, if the cost of 1 peach is 2 cents, as many peaches can be bought for 4 cents as 2 cents are contained times in 4 cents, which are 2. Therefore, at 2 cents each, 2 peaches can be bought for 4 cents.

LESSON XVI. — This form of analysis is to be used when the two given quantities are not of the same kind, and accords with the second part of the definition of Division given in Lesson XIV. The result must be of the same denomination as one of the given quantities, as in the case of the formula for multiplication (Lesson XI.). The remarks made upon that analysis will apply equally well here.

The expression "one half of 6 chestnuts" is not here used in a fractional sense, but is adopted as the most convenient mode of expressing the division of 6 chestnuts by the abstract number 2. In general, the division of a concrete by an abstract number is expressed by the same language that is employed in multiplying by a fraction whose numerator is unity. No division of the unit has yet been attempted, and consequently no real fraction has yet been introduced.

As Division is the reverse of Multiplication, it is evident that there may be two kinds. The product and multiplicand being given, to find the multiplier, corresponds with the first; while the product and multiplier being given, to find the multiplicand, corresponds with the second. Thus, 2 is contained in 6, 3 times, because 3 times 2 are 6; but one half of 6 is 3, because 2 times 3 are 6. In the first case, the *size* of the parts is given, to find their *number*; in the second, the *number* of parts is given, to find their *size*,

Ex. 23, 24. — These questions may be answered directly from the multiplication-table, or according to either of the following forms : —

99 are 9 times one ninth of 99, which is 11. Or,

Since 9 times some number are 99, once that number, or the number itself, must be one ninth of 99, which is 11.

The last form corresponds with that given at the commencement of this Lesson. Where fractions are introduced, as in Lesson XXVI., such questions cannot be answered directly by the multiplication-table, and a form like one of the above must be introduced; hence it may be convenient to use the same here.

In the last part of this Lesson, both kinds of division will be

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found, and the pupil will be required to determine which form of analysis is the proper one to employ. He will find no difficulty, however, if required to give the *denominations.* He will at once see the impropriety of attempting to find how many times 3 heifers are contained in 48 dollars, or of calling 1 tenth of 60 dollars 6 tons.

LESSON XVII. — The three formulas found in Lessons XI., XIV., and XVI. (those for multiplication, first division, and second division) are of such importance that they may appropriately be called the *fundamental formulas* of analysis. Nearly every operation which is hereafter introduced, excepting mere addition and subtraction, consists of an application of them. Each of the practical examples of this Lesson, after the first 18, requires the application of two of these formulas, the first one in each solution being that for multiplication.

Ex. 56 – 59. — These can readily be solved by either multiplication and first division, or multiplication and second division, according as the number of men, or the time, is reduced to unity. That solution is to be preferred which employs second division, and makes the intermediate result of the same kind as the answer.

Ex. 60. — The following is also a correct analysis : —

If 1 pine-apple is worth 2 oranges, 4 pine-apples will be worth 4 times 2 oranges, which are 8 oranges ; and if 1 orange is worth 3 apples, 8 oranges, or 4 pine-apples, will be worth 8 times 3 apples, which are 24 apples.

If the teacher desires to do so, he may require the pupil to repeat the conclusion, in connection with the conditions, for every example, as in the analysis of Ex. 19. The abbreviated forms of analysis have generally been used in the text.

LESSON XVIII. — This Lesson also contains examples which require the application of the three fundamental formulas.

Ex. 25. — The following form of analysis may also be employed : —

As 6 copies are contained in 12 copies 2 times, 12 copies will cost 2 times as much as 6 copies ; then, if 6 copies cost 8 dollars, 12 copies will cost 2 times 8 dollars, which are 16 dollars.

If, in this class of examples, the number connected with the required part be contained an exact number of times in the other given number of the same kind, as, for instance, if 12 copies be changed to 3 copies in Example 25, the following solution might be employed : —

As 3 copies are contained in 6 copies 2 times, 3 copies will cost one half as much as 6 copies ; then, if 6 copies cost 8 dollars, 3 copies will *cost one half of* 8 dollars, or 4 dollars.

The following form would correspond with that given in the text: —

If 6 copies cost 8 dollars, 3 copies will cost the same part of 8 dollars that 3 copies are of 6 copies, or one half; therefore 3 copies will cost one half of 8 dollars, which is 4 dollars.

Ex. 29. — This may also be solved by two applications of the first form of division, thus: —

If 4 cherries are worth 1 plum, 24 cherries will be worth as many plums as 4 cherries are contained times in 24 cherries, which are 6; and if 3 plums are worth 1 peach, 6 plums will be worth as many peaches as 3 plums are contained times in 6 plums, which are 2.

LESSON XIX. — Some teachers prefer the following forms of analysis: —

Ex. 1. — Since there are 10 mills in 1 cent, the number of mills is equal to 10 times the number of cents; 10 times 9 are 90. Therefore, in 9 cents there are 90 mills.

Ex. 2. — Since in 1 cent there are 10 mills, the number of cents is equal to one tenth of the number of mills; one tenth of 80 is 8. Therefore, in 80 mills there are 8 cents.

If the pupil were acquainted with fractions, the following analysis might also be used: —

Since 10 mills make 1 cent, 1 mill is 1 tenth of a cent, and 80 mills are 80 times 1 tenth of a cent, which are 80 tenths of a cent, or 8 ° cents.

LESSON XXII. Ex. 8. — The analysis may be thus expressed: —

A field 1 rod long and 1 rod wide contains 1 square rod; hence a field 12 rods long and 1 rod wide contains 12 times 1 square rod, which are 12 square rods, and a field 12 rods long and 9 rods wide contains 9 times 12 square rods, which are 108 square rods.

LESSON XXV. — The two following forms of analysis correspond with those given in Reduction (Lesson XIX.).

Ex. 4. — Since there are 2 halves in 1, the number of halves is equal to 2 times the number of whole ones; 2 times 2 are 4. Therefore, in 2 there are 4 halves.

Ex. 8. — Since in 1 there are 2 halves, the number of whole ones is equal to 1 half of the number of halves; 1 half of 6 is 3. Therefore, in 6 halves there are 3 whole ones.

Ex. 61. — ABBREVIATED ANAL. 3 is 1 half of 2 times 3, or 6.

LESSON XXVI. — In this Lesson the pupil is taught to divide one whole number by another, when the result involves a fraction. Heretofore, when the quotient has not been exact, he

has named the remainder, as such; now he is to complete the division of the dividend by dividing that remainder by the divisor. In order to do this, he must learn how to divide a less number by a greater.

In the first form of division, a new expression is introduced when the divisor is greater than the dividend. As we know that 2 cannot contain 6 any number of *times*, we inquire what *part* 2 is of 6. As a result, we state that 2 is the third part of 6, or one third of 6, instead of saying that 6 is contained in 2 one third of a time. This form of expression may also be employed when the divisor is less than the dividend, provided the quotient is given wholly in the fractional form.

In the second form of division (Ex. 48 – 70), the same expression is used in all cases, whether the divisor is less or greater than the dividend, and whether the quotient is to be wholly in the fractional form, or not.

Ex. 10. — The analysis given may be considered as the form which first division (Lesson XIV.) takes when the divisor is greater than the dividend. The following form of analysis may also be employed : —

As $2 are contained in $10, 5 times, $2 will buy 1 fifth as much as $10; then, if $10 buy 1 barrel, $2 will buy 1 fifth of a barrel.

Ex. 26. — The division may be represented thus : —

$8 = 6 + 2$; 3 is contained in 6, 2 times, and 2 is 2 thirds of 3. Therefore 8 are 2 times 3, and 2 thirds of 3.

Ex. 51. As it is impossible to divide 5 into 7 equal parts without dividing the units, we may take 1 seventh of each of the 5 apples; but, if the apples are of equal size, we shall then have the same amount that we should obtain by taking 5 sevenths of 1 apple.

Ex. 25 and 51. — After the pupil understands fully the reasoning involved, he may be allowed to answer such questions directly, thus: 3 apples are 3 sevenths of 7 apples. 1 seventh of 5 apples is 5 sevenths of an apple.

Ex. 57. — This division may be represented thus : —

$5 = 3 + 2$; 1 third of 3 is 1, and 1 third of 2 is 2 thirds. Therefore 1 third of 5 is 1 and 2 thirds.

This example may also be solved precisely like the 48th, and the result thus obtained can be easily reduced to the same form as that obtained by the other process, thus: —

1 third of 5 is 5 thirds, and since there is 1 whole one in 3 thirds, there are as many whole ones in 5 thirds as 3 thirds are contained times *in 5 thirds, which* are 1 and 2 thirds.

Ex. 61.—Here we may, if we choose, take 1 eighth of each of the 21 apples, as in Ex. 51; but we are not obliged to do so. We may make 8 parts of 2 apples each, and then there are only 5 apples left, which may be divided into 8 equal parts as in Ex. 51.

Ex. 63. ABBREVIATED ANAL. 5 is 2 times 1 half of 5, which is 2 and 1 half. (Ap., Les. XVI., Ex. 23, 24.)

LESSON XXVII. — The pupil will use the forms of analysis found in the last two Lessons.

Ex. 18. — We first find that 14 is 7 eighths of 16, and the question then becomes, 16 are how many times 4 ? The final result is stated thus: 14 is 7 eighths of 4 times 4.

Ex. 28.—Here the second form of division takes the place of the first form of division found in the preceding examples.

LESSON XXIX. Ex. 15.—Since in 1 there are $\frac{4}{4}$, in 5 there are 5 times $\frac{4}{4}$, which are $\frac{20}{4}$, or 20 times $\frac{1}{4}$.

This is simply reducing 5 to fourths.

Ex. 25. — Since 1 is equal to $\frac{4}{4}$, there are as many times 1 in $\frac{11}{4}$ as $\frac{4}{4}$ are contained times in $\frac{11}{4}$, which are $2\frac{3}{4}$. Therefore there are 2 times 1, and $\frac{3}{4}$ of 1, in $\frac{11}{4}$.

This is simply reducing $\frac{11}{4}$ to a mixed number.

Ex. 51. — A form of analysis somewhat similar to that given in Ap., Les. XXV. may be employed : —

Since 1 is equal to $\frac{3}{3}$, or $\frac{6}{6}$, and since 3 is $\frac{1}{2}$ of 6, the number of thirds is equal to $\frac{1}{2}$ of the number of sixths; $\frac{1}{2}$ of 4 is 2. Therefore $\frac{4}{6} = \frac{2}{3}$.

This is very nearly the same as the reasoning involved in applying the principle contained in the Note, which may be expressed as follows: —

Since the required denominator, 3, is $\frac{1}{2}$ of the given denominator, 6, we must also take $\frac{1}{2}$ of the numerator; $\frac{1}{2}$ of 4 is 2. Therefore $\frac{4}{6} = \frac{2}{3}$.

Ex. 53. — The forms of analysis given cannot readily be applied in such a case, because we do not know what denominator the fraction is to have when reduced to its lowest terms. The pupil can readily learn by trial what number is contained in both terms of a fraction. If there are two or more such numbers, he may have to reduce the fraction a second or third time before it reaches its lowest terms, unless he uses the *largest* number at first.

Ex. 70. — The remarks and formulas found under Ex. 51 apply equally well here, if care is taken to make the proper changes in the latter.

Since 1 is equal to $\frac{3}{3}$, or $\frac{6}{6}$, and since 6 is 2 times 3, the number of sixths is equal to 2 times the number of thirds; 2 times 2 are 4. Therefore $\frac{2}{3} = \frac{4}{6}$. Or,

Since the required denominator, 6, is 2 times the given denominator, 3, we must also take 2 times the numerator; 2 times 2 are 4. Therefore $\frac{2}{3} = \frac{4}{6}$.

LESSON XXXII. Ex. 1. — The last step of the analysis may be expressed thus : —

In $\frac{12}{6}$ of a barrel there are as many whole barrels as $\frac{6}{6}$ are contained times in $\frac{12}{6}$, which are $2\frac{2}{6}$.

Ex. 5. 10 times 3 = 30, and 10 times $\frac{1}{2} = \frac{10}{2} = 5$; $30 + 5 = 35$. Or, $3\frac{1}{2} = \frac{7}{2}$, and 10 times $\frac{7}{2} = \frac{70}{2} = 35$.

The former method is preferable for large numbers, the latter may be used for small ones.

LESSON XXXIII. Ex. 11. — The analysis may also be expressed thus : —

If 1 pound of butter cost 17 cents, $\frac{2}{6}$ of a pound will cost $\frac{2}{6}$ of 17 cents; $\frac{1}{6}$ of 17 cents is $\frac{17}{6}$ of a cent, and $\frac{2}{6}$ are 2 times $\frac{17}{6}$ of a cent, which are $\frac{34}{6}$ of a cent, or $6\frac{4}{6}$ cents.

Ex. 14. — Instead of saying "three times 12 cents, and $\frac{3}{4}$ of 12 cents," we may, for the sake of brevity, say "$3\frac{3}{4}$ times 12 cents." The mixed number may be reduced to an improper fraction before multiplying; but if large numbers are involved, it is not advisable to do so.

Ex. 16. — After the pupil is perfectly familiar with the analysis, it may be abbreviated thus : —

As $\frac{7}{8}$ of the cost was $ 33, the cost itself was $\frac{8}{8}$ of $ 33; $\frac{1}{8}$ of $ 33 is $ 4\frac{5}{8}$, and $\frac{8}{8}$ are 6 times $ 4\frac{5}{8}$, which are $ 28\frac{2}{8}$.

Ex. 19. — Mixed numbers must be reduced to improper fractions before we can divide by them.

Ex. 21. — We may either give the full reasoning, as in Ex. 48, Les. XXVI., or state the result at once, and reduce to the lowest terms. The last four results may also be obtained by actual *division, as in Ex. 57* of the same Lesson.

Ex. 31. 6 is $\frac{4}{4}$ of $\frac{4}{4}$ of 6. (See Ex. 61, Les. XXV., and Ex. 63, Les. XXVI.)

LESSON XXXIV. Ex. 6.—The following analysis may appear clearer to some than the one given in the text:—

If ½ of a melon was divided into 3 equal parts, ½, or the whole melon, would contain 4 times 3, or 12, such parts; therefore each one of the parts was $\frac{1}{12}$ of the melon.

After the pupil is familiar with the reasoning involved in either form of analysis, he may be allowed to obtain the result at once by multiplying the denominators together, thus:—

½ of ¼ of a melon is $\frac{1}{12}$ of a melon.

LESSON XXXVI. Ex. 1.—After the numbers are reduced to a common denominator, the second analysis of Ex. 1, Les. XXXV. may be employed, or the same, in substance, may be used without such reduction, thus:—

If a man walk ⅘ of a mile in 1 hour, he will walk 1 mile in ⅝ of an hour, and he will walk 2¼ of a mile in 2¼ of ⅝ of an hour, or 3⅛ hours.

LESSON XLI. Ex. 17.—After the pupil has used this analysis so often that he has fully mastered it, he may pass at once from the fraction to the corresponding per cent., from memory.

Ex. 18.—The following form of analysis would be convenient where there are no common factors in the given numbers, or where the basis of percentage is contained an exact number of times in 100:—

Since 10 is 100 per cent. of itself, 1 will be $\frac{1}{10}$ of 100 per cent., or 10 per cent., of 10, and 2 must be 2 times 10 per cent., or 20 per cent., of 10.

LESSON XLII. Ex. 15.—The analysis given in the text proceeds upon the supposition that 1 per cent. is simply $\frac{1}{100}$, rather than 1 on 100. In this example, we inquire, "40 cents are how many hundredths of 60 cents?" rather than, "a gain of 40 cents on 60 cents would be how many cents on 100 cents, or $1?" If the analysis were based upon the latter definition, it would be as follows:—

If 60 cents gain 40 cents, 1 cent would gain $\frac{1}{60}$ of 40 cents, or ⅔ of a cent, and 100 cents, or $1, would gain 100 times ⅔ of a cent, or 66⅔ cents.

Other analyses given in the text would need a corresponding change to conform to the second definition given above.

Ex. 25.—The difficulty in this and most of the succeeding

GENERAL REMARKS.

The three fundamental formulas (Ap., Les. XVII.) sustain a peculiar complementary relation to each other, as each of the first two, at least, is equivalent to the other two. Take, for instance, the simplest possible cases of each.

Ex. 1, Les. XI. may be analyzed thus: —

Since 2 cents will buy 1 apple, 1 cent will buy $\frac{1}{2}$ of an apple, and 2 apples will cost as many cents as $\frac{1}{2}$ of an apple is contained times in 2 apples, which are 4.

Here second division and first division produce the same result as multiplication.

Ex. 1, Les. XIV. may be analyzed thus: —

Since 2 cents will buy 1 peach, 1 cent will buy $\frac{1}{2}$ of a peach, and 4 cents will buy 4 times $\frac{1}{2}$ of a peach, which are 2 peaches.

Here second division and multiplication produce the same result as first division, and some prefer this analysis to the one given in the text.

The third formula may also be made equivalent to the other two by introducing an hypothesis not directly suggested by the question. Ex. 1, Les. XVI. may be analyzed thus: —

If he had given each brother 1 chestnut, 2 brothers would have received 2 times 1 chestnut, or 2 chestnuts; and if 2 chestnuts would supply 1 to each brother, 6 chestnuts would supply as many to each brother as 2 chestnuts are contained times in 6 chestnuts, which are 3.

Here multiplication and first division produce the same result as second division.

If, then, the simplest of examples may be analyzed in two different ways, it will at once be seen that the more complicated ones will admit of a still greater variety. Take, for instance, the simple question: —

If 3 apples cost 4 cents, what will 5 apples cost?

These five combinations of the given numbers will produce the required result, viz. $5 \times \frac{4}{3}$, $5 \div \frac{3}{4}$, $4 \times \frac{5}{3}$, $4 \div \frac{3}{5}$, and $\frac{5 \times 4}{3}$.

and there is an analysis corresponding with each of these. The first will combine second division and multiplication, like Ex. 1, Les. XVIII., the second will combine second division and first division, like Ex. 18, Les. XVIII., the third will combine first division and multiplication, like Ex. 25, Les. XVIII., .the fourth will combine first division and second division, and the fifth will combine multiplication and second division, as briefly indicated below.

1. ANAL. BY UNITY. — Since 3 apples cost 4 cents, 1 apple costs $\frac{1}{3}$ of 4 cents, or $\frac{4}{3}$ of a cent, and 5 apples cost 5 times $\frac{4}{3}$ of a cent, or $\frac{20}{3}$ of a cent.

2. ANAL. BY UNITY. — Since 4 cents buy 3 apples, 1 cent buys $\frac{1}{4}$ of 3 apples, or $\frac{3}{4}$ of an apple, and 5 apples cost as many cents as $\frac{3}{4}$ of an apple is contained times in 5 apples, which are $\frac{20}{3}$.

3. ANAL. BY RATIO. 5 apples are $\frac{5}{3}$ of 3 apples, and if 3 apples cost 4 cents, 5 apples cost $\frac{5}{3}$ of 4 cents, or $\frac{20}{3}$ of a cent.

4. ANAL. BY RATIO. 3 apples are $\frac{3}{5}$ of 5 apples, and if 3 apples cost 4 cents, then 4 cents are $\frac{3}{5}$ of the cost of 5 apples, and the cost itself is $\frac{5}{3}$ of 4 cents, or $\frac{20}{3}$ of a cent.

5. ANAL. BY HYPOTHESIS. — If 1 apple had cost 4 cents, then 5 apples would have cost 5 times 4 cents, or 20 cents; but as 3 apples cost 4 cents, 5 apples must cost $\frac{1}{3}$ of 20 cents, or $\frac{20}{3}$ of a cent.

The same methods of analysis apply to any similar example, but sometimes one, and sometimes another, will be preferable, according to the relations of the numbers involved. The last method generally supposes a reduction to unity, and is therefore more or less intimately connected with the first.
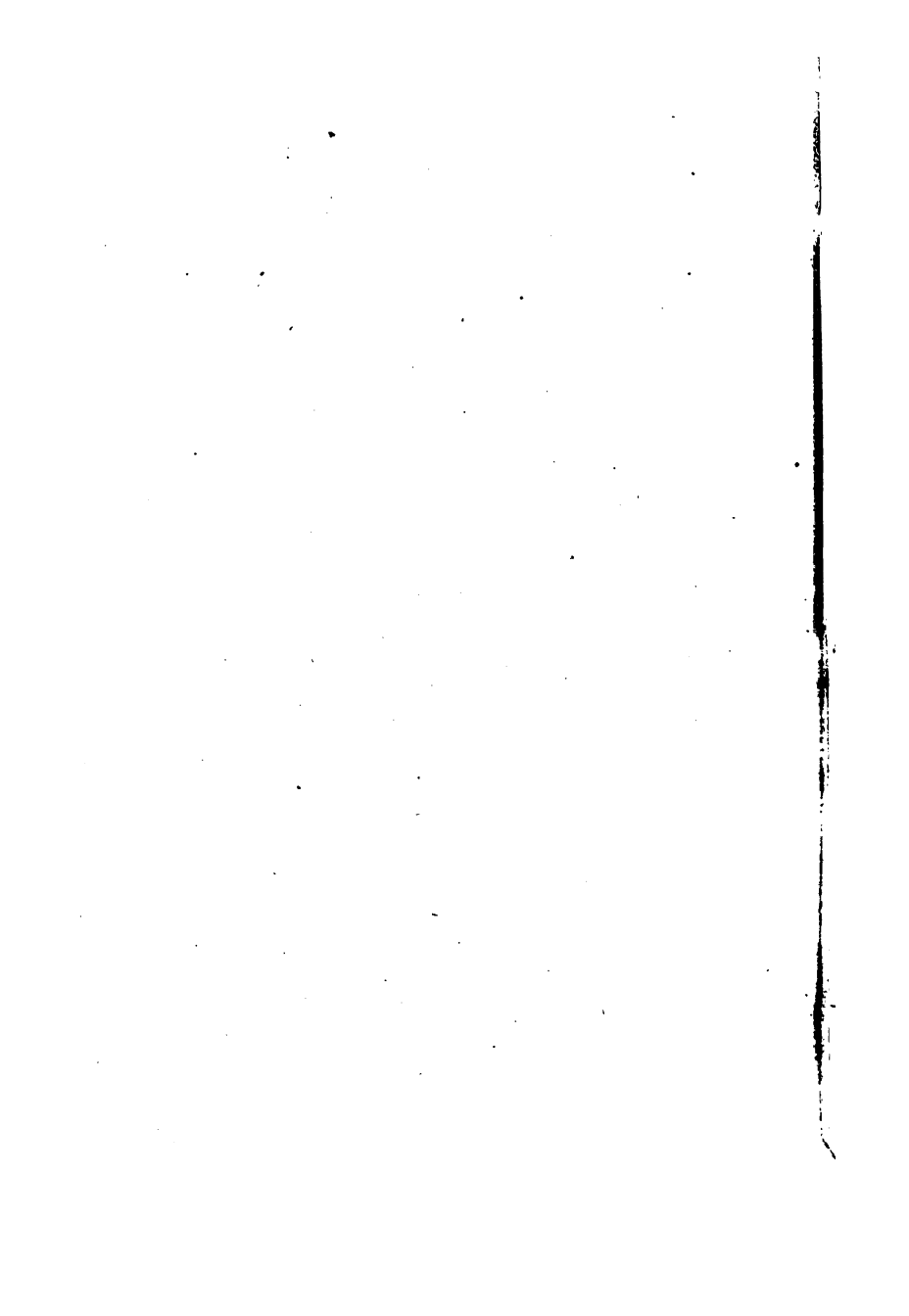
Once	2 times	3 times	4 times	5 times	6 times
1 is 1	1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 " 2	2 " 4	2 " 6	2 " 8	2 " 10	2 " 12
3 " 3	3 " 6	3 " 9	3 " 12	3 " 15	3 " 18
4 " 4	4 " 8	4 " 12	4 " 16	4 " 20	4 " 24
5 " 5	5 " 10	5 " 15	5 " 20	5 " 25	5 " 30
6 " 6	6 " 12	6 " 18	6 " 24	6 " 30	6 " 36
7 " 7	7 " 14	7 " 21	7 " 28	7 " 35	7 " 42
8 " 8	8 " 16	8 " 24	8 " 32	8 " 40	8 " 48
9 " 9	9 " 18	9 " 27	9 " 36	9 " 45	9 " 54
10 " 10	10 " 20	10 " 30	10 " 40	10 " 50	10 " 60
11 " 11	11 " 22	11 " 33	11 " 44	11 " 55	11 " 66
12 " 12	12 " 24	12 " 36	12 " 48	12 " 60	12 " 72

7 times	8 times	9 times	10 times	11 times	12 times
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 14	2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 21	3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 28	4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 35	5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 42	6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
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12 " 84	12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

13 times	14 times	15 times	16 times	17 times	18 times
1 are 13	1 are 14	1 are 15	1 are 16	1 are 17	1 are 18
2 " 26	2 " 28	2 " 30	2 " 32	2 " 34	2 " 36
3 " 39	3 " 42	3 " 45	3 " 48	3 " 51	3 " 54
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19 times	20 times	21 times	22 times	23 times	24 times
1 are 19	1 are 20	1 are 21	1 are 22	1 are 23	1 are 24
2 " 38	2 " 40	2 " 42	2 " 44	2 " 46	2 " 48
3 " 57	3 " 60	3 " 63	3 " 66	3 " 69	3 " 72
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12 " 228	12 " 240	12 " 252	12 " 264	12 " 276	12 " 288

13 times	14 times	15 times	16 times	17 times	18 times
13 are 169	13 are 182	13 are 195	13 are 208	13 are 221	13 are 234
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24 " 312	24 " 336	24 " 360	24 " 384	24 " 408	24 " 432

19 times	20 times	21 times	22 times	23 times	24 times
13 are 247	13 are 260	13 are 273	13 are 286	13 are 299	13 are 312
14 " 266	14 " 280	14 " 294	14 " 308	14 " 322	14 " 336
15 " 285	15 " 300	15 " 315	15 " 330	15 " 345	15 " 360
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